Chapter 13: Oscillations About Equilibrium

Answers to Even-Numbered Conceptual Questions

2. The person’s shadow undergoes periodic motion, with the same period as the period of the Ferris wheel’s rotation. In fact, if we take into account the connection between uniform circular motion and simple harmonic motion, we can say that the shadow exhibits simple harmonic motion as it moves back and forth on the ground.

4. Recall that the maximum speed of a mass on a spring is \( v_{\text{max}} = \omega A \), where \( \omega = \sqrt{k/m} \). It follows that the maximum kinetic energy is \( K_{\text{max}} = \frac{1}{2}mv_{\text{max}}^2 = \frac{1}{2}m(\omega A)^2 = \frac{1}{2}kA^2 \). Note that the mass cancels in our final expression for the maximum kinetic energy. Therefore, the larger mass moves more slowly by just the right amount so that the kinetic energy is unchanged.

6. The constant \( A \) represents the amplitude of motion; the constant \( B \) is the angular frequency. Noting that the angular frequency is \( \omega = 2\pi f \), we have that the frequency is \( f = \frac{\omega}{2\pi} = \frac{B}{2\pi} \).

8. The period of a pendulum is independent of the mass of its bob. Therefore, the period should be unaffected.

Solutions to Problems and Conceptual Exercises

1. Picture the Problem: The sketch shows the cart traveling up and back, thus completing one cycle on the track.

   Strategy: One period is the time for the cart to move down the track and back. The frequency is the inverse of the period.

   Solution: 1. Divide twice track length by the velocity to obtain the period:
   \[
   T = \frac{2(5.0 \text{ m})}{0.85 \text{ m/s}} = 11.8 \text{ s} = 12 \text{ s}
   \]

   2. Invert the period to determine the frequency:
   \[
   f = \frac{1}{T} = \frac{1}{11.8 \text{ s}} = 0.085 \text{ Hz}
   \]

   Insight: The full period of oscillation is complete when the cart returns to its starting position with the same speed and direction of motion with which it began.

2. Picture the Problem: The rocking chair completes one full cycle or oscillation each time it returns back to its original position.

   Strategy: The period is the time for one cycle. The frequency is the inverse of the period, or the number of cycles per second.

   Solution: 1. Divide the total time by the number of cycles to determine the period:
   \[
   T = \frac{t}{n} = \frac{21 \text{ s}}{12 \text{ cycles}} = 1.75 \text{ s}
   \]

   2. Invert the period to determine the frequency:
   \[
   f = \frac{1}{T} = \frac{1}{1.75 \text{ s}} = 0.57 \text{ Hz}
   \]

   Insight: Since period and frequency are inverses of each other, when the period is greater than a second, the frequency will be less than a hertz.
3. **Picture the Problem**: As the bobber moves up and down its motion is periodic. One period is the amount of time for the bob to drop down and rise back up to its original position.

   **Strategy**: We can find the period by taking the inverse of the period.

   **Solution**: Invert the period to obtain the frequency:
   \[ T = \frac{1}{f} = \frac{1}{2.6 \text{ Hz}} = 0.38 \text{ s} \]

   **Insight**: Because frequency is the inverse of period, when the frequency is greater than one hertz, the period will necessarily be less than a second.

4. **Picture the Problem**: As the basketball is dribbled it moves up and down in periodic motion. One period is the time for the ball to drop and return to the player’s hand.

   **Strategy**: The time for one dribble is the period, or the inverse of the frequency. Multiplying the period by the number of dribbles will give the total time.

   **Solution**: Multiply the inverse of the frequency by the number of dribbles to obtain the total time:
   \[ t = 12T = \frac{12}{f} = \frac{12}{1.77 \text{ Hz}} = 6.78 \text{ s} \]

   **Insight**: The greater the frequency, the more dribbles possible over a given time.

5. **Picture the Problem**: A heart beats in a regular periodic pattern. To measure your heart rate you typically count the number of pulses in a minute.

   **Strategy**: Convert the time from minutes to seconds to obtain the frequency in hertz. The period is obtained from the inverse of the frequency.

   **Solution**: 1. Multiply the heart rate by the correct conversion factor to get the frequency in Hz.
   \[ f = \left( \frac{74 \text{ beats}}{\text{min}} \right) \left( \frac{1 \text{ min}}{60 \text{ s}} \right) = 1.2 \text{ Hz} \]

   2. Invert the frequency to obtain the period:
   \[ T = \frac{1}{f} = \frac{1}{1.23 \text{ Hz}} = 0.81 \text{ s} \]

   **Insight**: Typical resting heartbeats have frequencies around one hertz and periods of about one second.

6. **Picture the Problem**: When you measure your pulse you typically count the number of pulses per minute. The number of beats per minute, or frequency, can change depending on whether you are resting or exercising.

   **Strategy**: Convert the time rate from per second to per minute to obtain the frequency in beats/min.

   **Solution**: 1. (a) Multiply the frequency by the conversion factor to obtain the heart rate:
   \[ \left( \frac{1.45 \text{ beats}}{\text{s}} \right) \left( \frac{60 \text{ s}}{\text{min}} \right) = 87.0 \text{ beats/min} \]

   2. (b) The number of beats per minute will increase because more beats per second means more beats per minute.

   3. (c) Multiply the new frequency by the conversion factor to obtain the heart rate:
   \[ \left( \frac{1.55 \text{ beats}}{\text{s}} \right) \left( \frac{60 \text{ s}}{\text{min}} \right) = 93.0 \text{ beats/min} \]

   **Insight**: Increasing the frequency in any set of units (such as beat/sec or beat/min) will increase the frequency in any other set of units.
7. Picture the Problem: As gasoline burns inside the engine of a car, it causes the pistons to expand, which in turn causes the crankshaft to rotate. Increasing the gas in the engine (revving) causes the crankshaft to rotate faster. The frequency of the car’s engine is measured as the number of times the crankshaft rotates per minute.

Strategy: The frequency is given in units of rev/min which can be converted to hertz. The period can then be found by inverting the frequency. Reverse the process to convert a period back into a frequency in hertz.

Solution: 1. (a) Convert \( f \) to hertz:
\[
f = \left( \frac{2700 \text{ rev}}{\text{min}} \right) \left( \frac{1 \text{ min}}{60 \text{ s}} \right) = 45 \text{ Hz}
\]
2. Invert the frequency to obtain the period:
\[
T = \frac{1}{f} = \frac{1}{45 \text{ Hz}} = 0.022 \text{ s}
\]
3. (b) Invert the new period and convert seconds to minutes to obtain the rpm:
\[
f = \frac{1}{T} = \frac{1 \text{ rev}}{0.044 \text{ s}} \times \frac{60 \text{ s}}{\text{min}} = 1400 \text{ rpm}
\]
Insight: Since period and frequency are inverses of each other, a longer period resulted in a lower frequency.

8. Picture the Problem: A mass moves back and forth in simple harmonic motion with amplitude \( A \) and period \( T \).

Strategy: Use the principles of simple harmonic motion to answer the conceptual question.

Solution: 1. (a) Suppose the mass begins at one extreme of its displacement at \( t = 0 \). In the first half of its period \( T \) it will move through the equilibrium position and all the way to the other extreme of its displacement, a distance of \( 2A \). In the second half of its period it will return to its original position through another distance of \( 2A \), for a total distance traveled of \( 4A \).
2. (b) In time \( 2T \) the mass will move distance \( 8A \), and in time \( \frac{T}{2} \) the mass will move distance \( 2A \), for a total distance traveled of \( 10A \).

Insight: The distance traveled in each case is the same regardless of where the mass starts in its cycle.

9. Picture the Problem: A mass moves back and forth in simple harmonic motion with amplitude \( A \) and period \( T \).

Strategy: Use the principles of simple harmonic motion to answer the conceptual question.

Solution: 1. (a) Suppose the mass begins at one extreme of its displacement at \( t = 0 \). In the first half of its period \( T \) it will move through the equilibrium position and all the way to the other extreme of its displacement, a distance of \( 2A \). It therefore requires a time \( \frac{T}{4} \) for the mass to travel a distance of \( 2A \).
2. (b) The mass moves a distance \( A \) in one-quarter of a cycle. Using the above result we can see that the mass will move a distance \( 2A \) in half a cycle and a distance \( A \) in another quarter of a cycle, for a total time elapsed of \( \frac{3}{4} T \).

Insight: The distance traveled in each case is the same regardless of where the mass starts in its cycle.

10. Picture the Problem: The position of the mass oscillating on a spring is given by the equation of motion.

Strategy: The oscillation period can be obtained directly from the argument of the cosine function. The mass is at one extreme of its motion at \( t = 0 \), when the cosine is a maximum. It then moves toward the center as the cosine approaches zero. The first zero crossing will occur when the cosine function first equals zero, that is, after one-quarter period.

Solution: 1. (a) Identify \( T \) with the time \( 0.58 \text{ s} \):
\[
\cos \left( \frac{2\pi}{T} t \right) = \cos \left( \frac{2\pi}{0.58 \text{ s}} t \right), \text{ therefore } T = 0.58 \text{ s}.
\]
2. (b) Multiply the period by one-quarter to find the first zero crossing:
\[
t = \frac{1}{4} (0.58 \text{ s}) = 0.15 \text{ s}.
\]
Insight: A cosine function is zero at \( \frac{1}{4} \) and \( \frac{3}{4} \) of a period. It has its greatest magnitude at 0 and \( \frac{1}{2} \) of a period.
11. **Picture the Problem:** A mass on a spring oscillates with simple harmonic motion.

**Strategy:** The oscillation period can be obtained directly from the argument of the cosine function. The frequency is the inverse of the period. The mass is at one extreme of its motion at \( t = 0 \), when the cosine is a maximum. In part (c) the mass is at the point of interest when the cosine function is equal to \(-1\). This occurs one-half a period later.

**Solution:**

1. (a) Observe that the period is in the denominator in the argument of the cosine:

\[

t = \frac{2 \pi t}{T} \quad \text{and} \quad f = \frac{1}{T}.
\]

Since \( \cos \left( \frac{2 \pi t}{0.68} \right) = 0.68 \), therefore \( T = 0.68 \) s.

2. Invert the period to obtain the frequency:

\[

f = \frac{1}{0.68} \text{ s} = 1.5 \text{ Hz}
\]

3. (b) The time the mass is at \(-7.8\) cm is half a period:

\[

t = \frac{1}{2} \left( \frac{0.68 \text{ s}}{2} \right) = 0.34 \text{ s}
\]

**Insight:** This problem could also be solved by setting the motion equation equal to \( x = -7.8 \) cm and solving for the time: \( t = (0.68\text{ s})/2\pi \) \cos \left(-\left(7.8 \text{ cm}/7.8 \text{ cm}\right)\right) = 0.34 \text{ s}. However, the approach outlined above is simpler.

12. **Picture the Problem:** A position-versus-time plot for an object undergoing simple harmonic motion is shown in the figure.

**Strategy:** Use the principles of one-dimensional motion to determine the rankings of speed, velocity, and acceleration for each of the indicated points.

**Solution:**

1. (a) The magnitude of the slope of the \( x \)-versus-\( t \) graph is the speed of the object. The slope is zero at points C and F, so the speed is also zero there. The slope is maximum at points A and D, so the speed is maximum at those points. The speeds at points B and E are intermediate between maximum and zero. We arrive at the ranking for speeds: \( C = F < B = E < A = D \).

2. (b) The velocity is the same as the speed except we must take into account the direction of motion. The slope is negative between C and F, so the velocities are negative there. We therefore arrive at the ranking of velocities: \( D < E < C = F < B < A \).

3. (c) The acceleration is the rate of change of the velocity, and it is proportional to the curvature of the \( x \)-versus-\( t \) plot. The acceleration can be positive, negative, or zero. The curvature is negative between points A and D (velocity is decreasing) and positive to the right of D (velocity is increasing). The curvature, and the acceleration, must therefore be zero at points A and D. We arrive at the ranking of the accelerations: \( C < B < A = D < E < F \).

**Insight:** The magnitude of the acceleration is maximum when the speed is zero, and zero when the speed is maximum.

13. **Picture the Problem:** A mass on a spring oscillates with simple harmonic motion of amplitude \( A \) about the equilibrium position \( x = 0 \). Its maximum speed is \( v_{\text{max}} \) and its maximum acceleration is \( a_{\text{max}} \).

**Strategy:** Note the relationships between position, velocity, and acceleration for simple harmonic motion when answering the conceptual questions. Refer to Figures 13-6 and 13-7 for help if necessary.

**Solution:**

1. (a) At \( x = 0 \) the speed of the mass is its maximum value, \( v_{\text{max}} \).

2. (b) The acceleration of the mass at \( x = 0 \) is \( 0 \).

3. (c) The point \( x = A \) is the turning point, so the speed of the mass is \( 0 \) there.

4. (d) At the point \( x = A \) the acceleration is \( -a_{\text{max}} \) (see Figure 13-7).

**Insight:** Newton’s Second Law predicts the acceleration is zero at \( x = 0 \) because the force is zero there, and that the acceleration is maximum at \( x = A \) because the force is maximum there.

14. **Picture the Problem:** A mass oscillates on a spring with a period \( T = 0.73 \) s and amplitude \( A = 5.4 \) cm, starting at \( x = A \) at time \( t = 0 \).

**Strategy:** Because the mass starts at \( x = A \) at time \( t = 0 \), its displacement is described by a cosine function, \( x = A \cos(\omega t) \). Identify the constants \( A \) and \( \omega \) from the given data.

**Solution:**

1. Identify the amplitude as \( A \): \( A = 5.4 \) cm
2. Calculate \( \omega \) from the period:
\[
\omega = \frac{2\pi}{T} = \frac{2\pi \text{ rad}}{0.73 \text{ s}} = 8.6 \text{ rad/s}
\]

3. Substitute \( A \) and \( \omega \) into the displacement equation:
\[
x = (5.4 \text{ cm}) \cos (8.6 \text{ rad/s} \cdot t)
\]

**Insight:** A cosine function has a maximum amplitude at \( t = 0 \), but a sine function has zero amplitude at \( t = 0 \). That is why we chose a cosine function to describe the motion.

**Picture the Problem:** When two or more atoms are bound in a molecule they are separated by an equilibrium distance. If the atoms get too close to each other the binding force is repulsive. When the atoms are too far apart the binding force is attractive. The nature of the binding force therefore is to cause the atoms to oscillate about the equilibrium distance.

**Strategy:** Since the mass starts at \( x = A \) at time \( t = 0 \), this is a cosine function given by \( x = A \cos (\omega t) \). From the data given we need to identify the constants \( A \) and \( \omega \). A cosine function is at its maximum at \( t = 0 \), but a sine function equals zero at \( t = 0 \).

**Solution:**
1. (a) Identify the amplitude as \( A \):
   \[ A = 3.50 \text{ nm} \]
2. Calculate the angular frequency from the frequency:
   \[ \omega = 2\pi f = 4.00\pi \times 10^{14} \text{ s}^{-1} \]
3. Substitute the amplitude and angular frequency into the cosine equation:
   \[ x = (3.50 \text{ nm}) \cos (4.00\pi \times 10^{14} \text{ s}^{-1} \cdot t) \]
4. (b) It will be a sine function, \( x = A \sin (\omega t) \), because sine satisfies the initial condition of \( x = 0 \) at \( t = 0 \).

**Insight:** A cosine function has a maximum amplitude at \( t = 0 \). A sine function has zero amplitude at \( t = 0 \).

**Picture the Problem:** One period of oscillation is shown in the figure.

**Strategy:** Since the mass is at \( x = 0 \) at \( t = 0 \), this will be a sine function. Substitute the amplitude and period into the sine equation to determine the general equation of motion. Finally substitute in the specific times to determine the position at each time.

**Solution:**
1. Write the sine equation in terms of the given amplitude and period:
   \[ x = (0.48 \text{ cm}) \cos \left( \frac{2\pi}{T} t \right) \]
2. (a) Substitute \( t = T/8 \) into the sine equation and evaluate:
   \[ x = (0.48 \text{ cm}) \cos \left( \frac{2\pi}{T} \frac{T}{8} \right) = (0.48 \text{ cm}) \cos \left( \frac{\pi}{4} \right) = 0.34 \text{ cm} \]
3. (b) Substitute \( t = T/4 \) into the sine equation and evaluate:
   \[ x = (0.48 \text{ cm}) \cos \left( \frac{2\pi}{T} \frac{T}{4} \right) = (0.48 \text{ cm}) \cos \left( \frac{\pi}{2} \right) = 0.48 \text{ cm} \]
4. (c) Substitute \( t = T/2 \) into the sine equation and evaluate:
   \[ x = (0.48 \text{ cm}) \cos \left( \frac{2\pi}{T} \frac{T}{2} \right) = (0.48 \text{ cm}) \cos (\pi) = 0 \]
5. (d) Substitute \( t = 3T/4 \) into the sine equation and evaluate:
   \[ x = (0.48 \text{ cm}) \cos \left( \frac{2\pi}{T} \frac{3T}{4} \right) = (0.48 \text{ cm}) \cos \left( \frac{3\pi}{2} \right) = -0.48 \text{ cm} \]
6. (e) Sketch a plot with the four data points:

**Insight:** The sine curve has been included in the sketch of part (e) to show that the four positions are consistent with a sine function.
17. **Picture the Problem:** A mass is attached to a spring. The mass is displaced from equilibrium and released from rest. The spring force causes the mass to oscillate about the equilibrium position in harmonic motion.

**Strategy:** The period can be obtained directly from the argument of the cosine function. Substituting the specific time into the equation will yield the location at that time. Substituting in the new time will show that the mass is at the same location one period later.

**Solution:**

1. (a) Identify the period \( (T) \) from the cosine equation: \( x = A \cos \left[ \frac{2\pi}{T} t \right] \), so here \( T = \frac{0.88 \text{ s}}{} \).

2. (b) Substitute \( t = 0.25 \text{ s} \) into the equation and evaluate \( x \):

   \[
   x = (6.5 \text{ cm}) \cos \left( \frac{2\pi}{0.88 \text{ s}} 0.25 \text{ s} \right) = -1.4 \text{ cm}
   \]

3. (c) Substitute \( t = (0.25 \text{ s} + T) \) into the equation and factor:

   \[
   x = A \cos \left[ \frac{2\pi}{T} (0.25 \text{ s} + T) \right] = A \cos \left[ \frac{2\pi}{T} (0.25 \text{ s} + 2\pi) \right]
   \]

4. Drop the \( 2\pi \) phase shift, because \( \cos (x + 2\pi) = \cos (x) \):

   \[
   x = A \cos \left[ \frac{2\pi}{T} 0.25 \text{ s} \right]
   \]

5. Insert the numeric values:

   \[
   x = (6.5 \text{ cm}) \cos \left( \frac{2\pi}{0.88 \text{ s}} 0.25 \text{ s} \right) = -1.4 \text{ cm} \Rightarrow \text{same location}
   \]

**Insight:** Increasing the time by any multiple of the period increases the argument of the cosine function by the same multiple of \( 2\pi \), which has no effect upon the value of the cosine function.

18. **Picture the Problem:** A mass is attached to a spring. The mass is displaced from equilibrium and released from rest. The spring force causes the mass to oscillate about the equilibrium position in harmonic motion.

**Strategy:** Because the mass starts from rest at \( t = 0 \), the harmonic equation will be a cosine function. We will use the amplitude and period to determine the general equation, which can be evaluated for any specific time. During the first half of each period the mass will be moving in the negative \( x \) direction toward the minimum, and during the second half the mass will move in the positive direction back toward the maximum. Therefore, we can determine the direction of motion by finding in which half of a period the time is located.

**Solution:**

1. (a) Insert the period and amplitude to create the general harmonic equation:

   \[
   x = A \cos (\omega t) = A \cos \left( \frac{2\pi}{T} t \right)
   \]

2. Insert \( T = 3.35 \text{ s} \) and \( t = 6.37 \text{ s} \) into the general equation and evaluate the position:

   \[
   x = (0.0440 \text{ m}) \cos \left( \frac{2\pi}{3.35 \text{ s}} 6.37 \text{ s} \right) = 0.0358 \text{ m}
   \]

3. (b) Divide the time by one period to find the number of periods that have elapsed:

   \[
   N = \frac{t}{T} = \frac{6.37 \text{ s}}{3.35 \text{ s}} = 1.90 \text{ periods}
   \]

4. Because this is slightly less than two full periods, the mass is headed away from equilibrium and toward its maximum positive displacement. It is therefore moving in the positive \( x \) direction.

**Insight:** This problem could also be solved by inserting a time slightly later than \( t = 6.37 \text{ s} \) (such as \( t = 6.38 \text{ s} \)) and evaluating the position \( (x = 0.0363 \text{ m}) \) as in step 2. Since this result is greater than 0.0358 m, the mass must be moving in the positive \( x \) direction. You could also calculate the velocity \( v = -A \omega \sin (\omega t) \) = 0.0479 m/s. It must be moving in the positive \( x \) direction because the sign of the velocity is positive.
19. **Picture the Problem**: One period of a sine wave is shown in the figure with the time the position is greater than \( A/2 \) shaded gray.

**Strategy**: The position is equal to \( A/2 \) at two times during the period. The difference in those two times is the portion of the period for which the position is greater than \( A/2 \). We can find those times by evaluating when the sine function is equal to one-half.

**Solution**: 1. Set the harmonic equation equal to \( A/2 \) and cancel the amplitudes:

\[
A \sin \left( \frac{2\pi t}{T} \right) = \frac{A}{2}
\]

\[
\sin \left( \frac{2\pi t}{T} \right) = \frac{1}{2}
\]

2. Take the arcsine of both sides of the equation and solve for \( t \):

\[
\frac{2\pi t}{T} = \sin^{-1}\left( \frac{1}{2} \right) \Rightarrow t = \frac{T}{2\pi} \sin^{-1}\left( \frac{1}{2} \right)
\]

3. Evaluate using \( \sin^{-1}\left( \frac{1}{2} \right) = \frac{\pi}{6} \) and \( \frac{5\pi}{6} \):

\[
t = \frac{T}{2\pi} \left( \frac{\pi}{6} \right) \quad \text{and} \quad t = \frac{T}{2\pi} \left( \frac{5\pi}{6} \right) \Rightarrow t = \frac{T}{12} \quad \text{and} \quad \frac{5T}{12}
\]

4. Subtract the first time from the second to find the time span:

\[
\frac{5T}{12} - \frac{T}{12} = \frac{4T}{12} = \frac{T}{3}
\]

Therefore, the object’s position is greater than \( A/2 \) for \( \frac{1}{3} \) of a cycle.

**Insight**: If the problem had asked for the time over which the mass was displaced by a distance greater than \( A/2 \) from the origin, the answer would have been \( \frac{2}{3} \) of a cycle, because we would need to include the time the mass was located between \( -A/2 \) and \( -A \).

20. **Picture the Problem**: One velocity over one period is shown in the figure. The region for which the speed \(|v|\) is greater than \( v_{\text{max}}/2 \) is shaded gray.

**Strategy**: Since speed is the magnitude of the velocity it will be greater than \( v_{\text{max}}/2 \) twice during each period. The two regions are symmetric; the total time is double the time interval over which the velocity is positive. The end points of the regions are found when the sine function is equal to one-half.

**Solution**: 1. Set equation 13-6 equal to \( v_{\text{max}}/2 \) and divide out \( v_{\text{max}} \):

\[
v_{\text{max}} \sin \left( \frac{2\pi t}{T} \right) = \frac{1}{2} v_{\text{max}}
\]

\[
\sin \left( \frac{2\pi t}{T} \right) = \frac{1}{2}
\]

2. Take the arcsine of each side and solve for \( t \):

\[
\frac{2\pi t}{T} = \sin^{-1}\left( \frac{1}{2} \right) \Rightarrow t = \frac{T}{2\pi} \sin^{-1}\left( \frac{1}{2} \right)
\]

3. Evaluate using \( \sin^{-1}\left( \frac{1}{2} \right) = \frac{\pi}{6} \) and \( \frac{5\pi}{6} \):

\[
t = \frac{T}{2\pi} \left( \frac{\pi}{6} \right) \quad \text{or} \quad t = \frac{T}{2\pi} \left( \frac{5\pi}{6} \right) = \frac{T}{12} \quad \text{or} \quad \frac{5T}{12}
\]

4. Subtract the first time from the second time and multiply by two:

\[
2 \left( \frac{5T}{12} - \frac{T}{12} \right) = 2 \left( \frac{4T}{12} \right) = \frac{2T}{3}
\]

The mass’s speed is greater than \( v_{\text{max}}/2 \) for \( \frac{2}{3} \) of a cycle.

**Insight**: If the problem had asked for the time that the velocity was greater than \( +v_{\text{max}}/2 \), then only the crest of the cycle would have been included and the time would have been \( \frac{1}{3} \) of a cycle.
21. **Picture the Problem:** When an object is oscillating in simple harmonic motion it experiences a maximum acceleration when it is displaced at its maximum amplitude. As the object moves toward the equilibrium position, the acceleration decreases and the velocity of the object increases. The object experiences its maximum velocity as it passes through the equilibrium position.

**Strategy:** The maximum velocity and acceleration can both be written in terms of the amplitude and angular speed, \( v_{\text{max}} = A\omega, a_{\text{max}} = A\omega^2 \). We can rearrange these equations to solve for the amplitude and angular speed. Then we can use the angular speed to determine the period.

**Solution:**

1. (a) Divide the square of the velocity by the acceleration to find the amplitude:

   \[
   A = \frac{(A\omega)^2}{A\omega^2} = \frac{v_{\text{max}}^2}{a_{\text{max}}} = \frac{4.3 \text{ m/s}^2}{0.65 \text{ m/s}^2} = 28 \text{ m}
   \]

2. (b) Divide the acceleration by the velocity to determine the angular speed:

   \[
   \omega = \frac{A\omega^2}{A\omega} = \frac{a_{\text{max}}}{v_{\text{max}}} = \frac{2\pi v_{\text{max}}}{a_{\text{max}}}
   \]

3. Divide \( 2\pi \) by the angular speed to calculate \( T \):

   \[
   T = \frac{2\pi}{\omega} = \frac{2\pi}{\left(\frac{a_{\text{max}}}{v_{\text{max}}}\right)} = \frac{2\pi v_{\text{max}}}{a_{\text{max}}}
   \]

**Insight:** When two or more quantities are functions of the same variables, it is often possible to rearrange the equations to isolate one or more of the variables. This can be a useful mathematical procedure.

22. **Picture the Problem:** This figure shows a ball rolling on a circular track with angular speed \( \omega = 1.3 \text{ rad/s} \). At time \( t = 0 \) the ball is at the angle \( \theta = 0 \). At a later time the ball is at the angle \( \theta \).

**Strategy:** Since the ball is initially at the angle \( \theta = 0 \), it is initially at the maximum horizontal position. The general position can be written using the cosine equation with the amplitude equal to the radius of the circle. Substituting in the given times into the cosine equation we can calculate the ball’s horizontal positions.

**Solution:**

1. Substitute \( A \) and \( \omega \) into the cosine equation:

   \[
   x = A \cos(\omega t) = (0.62 \text{ m}) \cos[(1.3 \text{ rad/s}) t]
   \]

2. (a) Substitute \( t = 2.5 \text{ s} \) into the position equation:

   \[
   x = (0.62 \text{ m}) \cos[(1.3 \text{ rad/s}) 2.5 \text{ s}] = -0.62 \text{ m}
   \]

3. (b) Substitute \( t = 5.0 \text{ s} \) into the position equation:

   \[
   x = (0.62 \text{ m}) \cos[(1.3 \text{ rad/s}) 5.0 \text{ s}] = 0.61 \text{ m}
   \]

4. (c) Substitute \( t = 7.5 \text{ s} \) into the position equation:

   \[
   x = (0.62 \text{ m}) \cos[(1.3 \text{ rad/s}) 7.5 \text{ s}] = -0.59 \text{ m}
   \]

**Insight:** The \( x \)-component of the ball’s position exhibits the same simple harmonic motion that a mass on a spring would exhibit.

23. **Picture the Problem:** When an object is oscillating in simple harmonic motion it experiences a maximum acceleration when it is displaced at its maximum amplitude. As the object moves toward the equilibrium position the acceleration decreases and the velocity of the object increases. The object experiences its maximum velocity as it passes through the equilibrium position.

**Strategy:** The maximum velocity and acceleration can both be written in terms of the amplitude and angular speed, \( v_{\text{max}} = A\omega, a_{\text{max}} = A\omega^2 \). Rearrange these equations to solve for the amplitude and angular speed. Then use the angular speed to determine the period.

**Solution:**

1. (a) Divide the square of the velocity by the acceleration to find the amplitude:

   \[
   A = \frac{(A\omega)^2}{A\omega^2} = \frac{v_{\text{max}}^2}{a_{\text{max}}} = \frac{(4.3 \text{ m/s})^2}{(0.65 \text{ m/s}^2)} = 28 \text{ m}
   \]
2. (b) Divide the acceleration by the velocity to determine the angular speed:

\[ \omega = \frac{A\omega}{A\omega} = \frac{a_{\text{max}}}{v_{\text{max}}} \]

3. Divide \( 2\pi \) by the angular speed to calculate \( T \):

\[ T = \frac{2\pi}{\omega} = \frac{2\pi}{v_{\text{max}}} = \frac{2\pi v_{\text{max}}}{a_{\text{max}}} = \frac{2\pi (4.3 \text{ m/s})}{(0.65 \text{ m/s}^2)} = 42 \text{ s} \]

**Insight:** When two or more quantities are functions of the same variables, it is often possible to rearrange the equations to uniquely determine those variables.

24. **Picture the Problem:** As the child rocks back and forth on the swing, her speed increases as she approaches the equilibrium of the swing and then decreases back to zero at the end of the swing. The maximum speed occurs when the swing is vertical.

**Strategy:** The maximum velocity equals the amplitude times the angular speed, which in turn depends upon the period.

**Solution:**

1. Write the maximum velocity in terms of amplitude and period:

\[ v_{\text{max}} = A\omega = A\left(\frac{2\pi}{T}\right) = (0.204 \text{ m})(\frac{2\pi}{2.80 \text{ s}}) = 0.458 \text{ m/s} \]

2. Insert the amplitude and period into the equation for maximum speed:

\[ v_{\text{max}} = A\left(\frac{2\pi}{T}\right) = \frac{2\pi (0.204 \text{ m})}{2.80 \text{ s}} = 0.458 \text{ m/s} \]

**Insight:** The girl’s motion has an amplitude of 8.03 inches and a maximum speed of 1.02 mi/h. Not a very exciting swing.

25. **Picture the Problem:** As the goldfinch bobs up and down on the branch, the branch moves in periodic motion. The acceleration will be greatest at the top and bottom of the bob, when the branch is farthest from its equilibrium position. The speed of the finch will be greatest at the center of the bob, when the branch is at equilibrium.

**Strategy:** The maximum acceleration is equal to the amplitude times the angular speed squared. We can calculate the angular speed from the period. We can calculate the maximum speed from the amplitude and angular speed. The maximum speed occurs when the acceleration (change in speed) is zero. The inverse is also true, that the maximum acceleration occurs when the speed is zero (a minimum).

**Solution:**

1. (a) Write the maximum acceleration in terms of amplitude and period:

\[ a_{\text{max}} = A\omega^2 = A\left(\frac{2\pi}{T}\right)^2 \]

2. Insert amplitude and period into maximum acceleration equation:

\[ a_{\text{max}} = A\left(\frac{2\pi}{T}\right)^2 = (0.0335 \text{ m})(\frac{2\pi}{1.65 \text{ s}})^2 = 0.486 \text{ m/s}^2 \]

3. Factor out \( g = 9.81 \text{ m/s}^2 \):

\[ a_{\text{max}} = (0.486 \text{ m/s}^2)(\frac{g}{9.81 \text{ m/s}^2}) = 0.0495g \]

4. (b) Write the maximum velocity in terms of amplitude and period:

\[ v_{\text{max}} = A\omega = A\left(\frac{2\pi}{T}\right) \]

5. Insert amplitude and period into maximum velocity equation:

\[ v_{\text{max}} = A\left(\frac{2\pi}{T}\right) = (0.0335 \text{ m})(\frac{2\pi}{1.65 \text{ s}}) = 0.128 \text{ m/s} \]

6. The speed is a minimum when the acceleration is a maximum because the maximum acceleration occurs when the branch is farthest from the equilibrium point. At this turning point, the finch is at rest (speed equals zero).

**Insight:** The mass of the finch was not needed in solving this problem. However, the mass of the finch does affect the outcome, because it is a factor in determining the period. A larger bird would oscillate with a longer period, thereby resulting in smaller maximum acceleration and velocity.
26. **Picture the Problem:** The ends of the tuning forks vibrate rapidly to produce the sound. When the ends are farthest away from equilibrium they experience the greatest restoring force, and thus the greatest acceleration. The ends move the swiftest as they pass through the equilibrium position.  

**Strategy:** The maximum velocity and acceleration can both be written in terms of the amplitude and angular speed, \( v_{\text{max}} = A\omega, a_{\text{max}} = A\omega^2 \), where angular speed is \( 2\pi \) times the frequency.  

**Solution:**  
1. (a) Multiply the amplitude by \( \omega \) to find the maximum velocity:  
\[ v_{\text{max}} = A\omega = A(2\pi f) = (0.00125 \text{ m})(2\pi \times 128 \text{ Hz}) = 1.01 \text{ m/s} \]

2. (b) Multiply the amplitude by \( \omega^2 \) to find the maximum acceleration:  
\[ a_{\text{max}} = A\omega^2 = A(2\pi f)^2 = (1.25 \text{ mm})(2\pi \times 128 \text{ Hz})^2 = 808.5 \text{ m/s}^2 \]

3. Factor out \( g = 9.81 \text{ m/s}^2 \) to obtain the acceleration as a multiple of \( g \):  
\[ a_{\text{max}} = 808.5 \text{ m/s}^2 \left( \frac{g}{9.81 \text{ m/s}^2} \right) = 82.4g \]

**Insight:** Tuning forks typically have two tines which oscillate in opposite directions. The net acceleration of the entire fork is thus zero even though the tips are accelerating at a large rate.

27. **Picture the Problem:** A structural beam is a metal rod that is necessary to maintain the shape and integrity of the spacecraft. Large forces, which could be caused by small, but rapid oscillations, could damage the support beam, jeopardizing the integrity of the spacecraft.  

**Strategy:** The maximum acceleration can be written in terms of the amplitude and angular speed, \( a_{\text{max}} = A\omega^2 \), where angular speed is \( 2\pi \) times the frequency.  

**Solution:**  
1. Multiply the amplitude by the square of \( 2\pi \) times the frequency to get \( a_{\text{max}} \):  
\[ a_{\text{max}} = A\omega^2 = A(2\pi f)^2 = (0.25 \text{ mm})(2\pi \times 110 \text{ Hz})^2 = 119 \text{ m/s}^2 \]

2. Factor out \( g = 9.81 \text{ m/s}^2 \) to obtain the acceleration as a multiple of \( g \):  
\[ a_{\text{max}} = 119 \text{ m/s}^2 \left( \frac{g}{9.81 \text{ m/s}^2} \right) = 12g \]

**Insight:** Since the acceleration is proportional to the square of the frequency, a large frequency will result in a very large acceleration. This is true even for small amplitude oscillations.

28. **Picture the Problem:** The figure shows a turntable rotating with a peg on its outer rim. The table is illuminated on one side. The shadow of the peg moves with simple harmonic motion along the wall.  

**Strategy:** The shadow of the peg moves along the wall with simple harmonic motion. The period is the time for the peg (and as such the shadow) to complete a full revolution. The amplitude is the same as the radius of the circle, and is also the maximum distance from the center. Knowing the period and amplitude we can calculate the maximum velocity, remembering that the angular frequency is inversely related to the period. Finally, use the amplitude and period to calculate the maximum acceleration.  

**Solution:**  
1. (a) Divide the circumference of the turntable by the peg’s tangential velocity to get the period of rotation:  
\[ T = \frac{C}{v} = \frac{2\pi r}{v} = \frac{2\pi(0.23 \text{ m})}{0.77 \text{ m/s}} = 1.9 \text{ s} \]

2. (b) Set the amplitude equal to the radius of the turntable:  
\[ A = r = 0.23 \text{ m} \]

3. (c) Insert the period and amplitude into the maximum velocity acceleration:  
\[ v_{\text{max}} = A\omega = \frac{2\pi A}{T} = \frac{A v}{r} = v = \frac{0.77 \text{ m/s}}{0.23 \text{ m}} = 3.3 \text{ m/s} \]

4. (d) Insert the period and amplitude into the maximum acceleration equation:  
\[ a_{\text{max}} = A\omega^2 = A \left( \frac{2\pi}{T} \right)^2 = A \left( \frac{v}{r} \right)^2 = v^2 = \frac{(0.77 \text{ m/s})^2}{0.23 \text{ m}} = 2.6 \text{ m/s}^2 \]

**Insight:** The maximum speed of the shadow is equal to the tangential speed of the peg. The shadow and the peg travel at the same speed when the peg travels perpendicular to the light source. The shadow travels slower than the peg when a component of the peg’s velocity is parallel to the light.
29. **Picture the Problem:** In an engine the moving pistons compress the fuel in the chamber and expand after the fuel has been ignited. This motion provides the power to the car. The frequency of the piston motion is measured by the number of revolutions of the crankshaft per minute (rev/min).

**Strategy:** To solve for the maximum acceleration and speed we must first convert the angular speed from rev/min to rad/s. Then we can use $A$ and $\omega$ to find the maximum acceleration and the maximum speed.

**Solution:**
1. Convert the angular speed to rad/s:
   $$\left(\frac{1700 \text{ rev}}{\text{min}}\right) \left(\frac{2\pi \text{ rad}}{\text{rev}}\right) \left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 178 \text{ s}^{-1}$$

2. (a) Use the amplitude and angular speed to solve for maximum acceleration:
   $$a_{\text{max}} = A\omega^2 = 3.5 \text{ cm} \left(178 \text{ s}^{-1}\right)^2 = \frac{1.1 \text{ km/s}^2}{\text{rad}}$$

3. (c) Use the amplitude and angular speed to solve for maximum speed:
   $$v_{\text{max}} = A\omega = 3.5 \text{ cm} \left(178 \text{ s}^{-1}\right) = \frac{6.2 \text{ m/s}}{\text{rad}}$$

**Insight:** The maximum acceleration of the pistons is over 100 times the acceleration due to gravity. Therefore the gravitational force has negligible effect on the motion of a working piston.

30. **Picture the Problem:** An air cart is attached to the end of a spring allowed to oscillate. Its displacement from equilibrium is given by $x = (10.0 \text{ cm}) \cos \left(\frac{2.00 \text{ s}^{-1}}{\text{rad}}\right)t + \pi$.

**Strategy:** The maximum kinetic energy can be obtained by inserting the maximum velocity into the kinetic energy equation. The maximum force can be obtained by substituting the maximum acceleration into Newton’s Second Law. From the position equation we see that the amplitude is $A = 10.0 \text{ cm} = 0.100 \text{ m}$ and the angular speed is $\omega = 2.00 \text{ rad/s}$. We will use these values to calculate the maximum kinetic energy and force.

**Solution:**
1. (a) Write the kinetic energy in terms of mass, amplitude, and angular speed:
   $$K_{\text{max}} = \frac{1}{2} m v_{\text{max}}^2 = \frac{1}{2} m \left(A\omega\right)^2 = \frac{1}{2} m A^2 \omega^2$$

2. Insert the values of mass, amplitude, and angular speed to calculate $K_{\text{max}}$:
   $$K_{\text{max}} = \frac{1}{2} \left(0.84 \text{ kg}\right) \left(0.100 \text{ m}\right)^2 \left(2.00 \text{ rad/s}\right)^2 = 0.017 \text{ J}$$

3. (b) Write the force equation in terms of mass, amplitude, and angular speed:
   $$F_{\text{max}} = ma_{\text{max}} = m \left(A\omega^2\right)$$

4. Insert the values of mass, amplitude, and angular speed to calculate the maximum force:
   $$F_{\text{max}} = \left(0.84 \text{ kg}\right) \left(0.100 \text{ m}\right) \left(2.00 \text{ rad/s}\right)^2 = 0.34 \text{ N}$$

**Insight:** The phase shift of $+\pi$ in the displacement equation has no effect on either $K_{\text{max}}$ or $F_{\text{max}}$, but it reverses the sign of the displacement. Therefore, at $t = 0$ the cart starts $x = -10 \text{ cm}$ instead of at $x = 10 \text{ cm}$. 

31. **Picture the Problem:** The figure shows the rider on the mechanical horse.

**Strategy:** (a) The rider will separate from the mechanical horse if $a_{\text{max}}$ at the top of the motion exceeds the acceleration of gravity, $g$. Therefore, because $a_{\text{max}}$ is related to the angular speed and the amplitude, we can set $a_{\text{max}}$ equal to $g$ and solve for the amplitude.

**Solution:**
1. (b) Write $a_{\text{max}}$ in terms of $A$ and $\omega$ and set it equal to $g$:
   $$a_{\text{max}} = A\omega^2 = g$$

2. Write angular speed in terms of period and solve for amplitude:
   $$A \left(\frac{2\pi}{T}\right)^2 = g \Rightarrow A = \left(\frac{T}{2\pi}\right)^2 g$$

3. Insert the numeric values for the period and gravity:
   $$A = \left(\frac{0.74 \text{ s}}{2\pi}\right)^2 \left(9.81 \text{ m/s}^2\right) = 0.14 \text{ m}$$

**Insight:** As long as the maximum acceleration is less than gravity, the rider will remain in contact with the horse, because an upward normal force is required to support the rider. When the maximum acceleration is greater than gravity, the normal force is no longer necessary and the rider will accelerate up from the horse unless an additional downward force is applied to the rider.
32. **Picture the Problem**: If a mass $m$ is attached to a given spring, its period of oscillation is $T$. Two such springs are connected end-to-end and the same mass $m$ is attached to one end.

**Strategy**: Determine the effective force constant of the two springs connected in series and compare it with the force constant of a single spring. Then use the relationship between period and force constant to predict the effect upon $T$.

**Solution**: (a) Two springs connected in series will stretch twice as far as a single spring when the same force is applied. This means that the effective force constant of the two springs connected end-to-end must be half that of a single spring (see Problem 83 in Chapter 7). By examining equation 13-11, $T = 2\pi\sqrt{m/k}$, we can see that cutting $k$ in half will increase $T$ by a factor of $\sqrt{2}$. We conclude that the resulting period of oscillation is **greater than** $T$.

(b) The best explanation is **III**. The longer spring stretches more easily, and hence takes longer to complete an oscillation. Statement I is false and statement II is true, but irrelevant because the spring force constant really does change.

**Insight**: If the springs were connected in parallel, as in Problem 84 of Chapter 7, the effective force constant would be $2k$ and the oscillation period would decrease.

33. **Picture the Problem**: An old car with worn-out shock absorbers oscillates with a given frequency when it hits a speed bump. The driver adds a couple of passengers to the car and hits another speed bump.

**Strategy**: Note the relationship between frequency of a mass on a spring and the mass in order to answer the conceptual question.

**Solution**: (a) Rewrite equation 13-11 as $f = 1/T = (1/2\pi)\sqrt{k/m}$. From this expression we can see that increasing $m$ will decrease $f$. We conclude that the resulting frequency of oscillation is **less than** it was before.

(b) The best explanation is **I**. Increasing the mass on a spring increases its period, and hence decreases its frequency. Statements II and III are each false.

**Insight**: The shock absorbers are designed to damp out such oscillations for the comfort of the passengers.

34. **Picture the Problem**: If a mass $m$ is attached to a given spring, its period of oscillation is $T$. Two such springs are connected end-to-end and the same mass $m$ is attached to one end.

**Strategy**: Determine the effective force constants of the two springs connected in the arrangements shown in the figure. Then use the relationship between the period and the force constant to predict the effect upon $T$.

**Solution**: (a) Twice as much force is required to stretch two springs connected in parallel (as in block 1) than is required to stretch a single spring the same distance. However, block 2 experiences the very same restoring force as block 1 because whenever one spring is stretched, the other is compressed, and the two forces add to make a double force. This means that the effective force constants of the arrangements are identical. We conclude that the period of block 1 is **equal to** the period of block 2.

(b) The best explanation is **II**. The two blocks experience the same restoring force for a given displacement from equilibrium, and hence they have equal periods of oscillation. Statement I is true, but irrelevant because the springs for block 2 aren’t connected in series. Statement III is false because the forces add, they don’t cancel.

**Insight**: When two springs are truly connected in series they will stretch twice as far as a single spring when the same force is applied. This means their force constant is effectively half that of a single spring.

35. **Picture the Problem**: This is a dimensional analysis problem.

**Strategy**: The units can be obtained by replacing the variables $k$ and $m$ with their respective units (N/m and kg) in the radical and simplifying.

**Solution**: 1. Replace $k$ and $m$ with their respective units: $\sqrt{\frac{k}{m}} \rightarrow \frac{N/m}{kg} = \frac{N}{m \cdot kg}$

2. Replace $N$ with $kg \cdot m/s^2$ and simplify: $\sqrt{\frac{N}{m \cdot kg}} = \frac{kg \cdot m/s^2}{m \cdot kg} = \frac{1}{s^2} \times s^{-1}$
**Insight:** Dimensional analysis is a useful tool to verify that a combination of parameters results in the correct final parameter. For example, if on a test you can’t remember if frequency is related to $\sqrt{k/m}$ or $\sqrt{m/k}$ you can use dimensional analysis to find the right one (it turns out to be $\sqrt{k/m}$).

36. **Picture the Problem:** A mass attached to a spring is pulled slightly away from equilibrium and released. The mass then oscillates about the equilibrium position at a frequency determined by the stiffness of the spring.

**Strategy:** We can determine the spring force constant by solving the equation for the period of a mass on a spring for the spring force constant, and substituting in the given period and mass.

**Solution:**
1. Solve the period equation for the spring force constant:
   $$T = 2\pi \sqrt{\frac{m}{k}} \Rightarrow k = \left(\frac{2\pi}{T}\right)^2 m$$

2. Insert the numeric values for $T$ and $m$:
   $$k = \left(\frac{2\pi}{0.77\text{ s}}\right)^2 m = \left(\frac{2\pi}{0.77\text{ s}}\right)^2 (0.46\text{ kg}) = 31\text{ N/m}$$

**Insight:** Measuring the period of oscillation is in many cases the most accurate way of measuring a spring force constant.

37. **Picture the Problem:** Four mass/spring systems have different masses and spring force constants.

**Strategy:** Use the relationship between period, mass, and spring force constant (equation 13-11) to determine the ranking of the various systems.

**Solution:**
1. Use eq. 13-11 to find $T_A$ for system A:
   $$T_A = 2\pi \sqrt{\frac{m_A}{k_A}} = 2\pi \sqrt{\frac{m}{k}}$$

2. Repeat to find $T_B$:
   $$T_B = 2\pi \sqrt{\frac{m_B}{k_B}} = 2\pi \sqrt{\frac{2m}{k}} = \sqrt{2} T_A$$

3. Repeat to find $T_C$:
   $$T_C = 2\pi \sqrt{\frac{3m}{6k}} = \frac{1}{\sqrt{2}} T_A$$

4. Repeat to find $T_D$:
   $$T_D = 2\pi \sqrt{\frac{m_D}{k_D}} = 2\pi \sqrt{\frac{m}{4k}} = \frac{1}{2} T_A$$

5. By comparing the resulting periods we arrive at the ranking $D < C < A < B$.

**Insight:** Measuring the period of oscillation is in many cases the most accurate way of measuring a spring force constant.

38. **Picture the Problem:** The top image shows Block 1 with identical springs in the left side. The bottom image shows Block 2 with identical springs on each side.

**Strategy:** Use Newton’s Second Law for the force on the block to calculate the effective spring force constant. Then put this constant into the period equation. In both cases the spring force constants add together to produce the effective constant.

**Solution:**
1. (Block 1) Write out the horizontal component of Newton’s Second Law:
   $$\sum F_x = ma = -kx - kx = -2kx = -k_{\text{effective}}x$$

   $$k_{\text{effective}} = 2k$$

2. Substitute the effective spring force constant into the equation for the period:
   $$T = 2\pi \sqrt{\frac{m}{k_{\text{effective}}}} = 2\pi \sqrt{\frac{1.25\text{ kg}}{2(49.2\text{ N/m})}} = 0.708\text{ s.}$$

3. (Block 2) Write out the horizontal component of Newton’s Second Law:
   $$\sum F_x = ma = -kx - kx = -2kx = -k_{\text{effective}}x$$

   $$k_{\text{effective}} = 2k$$

4. Substitute the effective spring force constant into the equation for the period:
   $$T = 2\pi \sqrt{\frac{m}{k_{\text{effective}}}} = 2\pi \sqrt{\frac{1.25\text{ kg}}{2(49.2\text{ N/m})}} = 0.708\text{ s.}$$

**Insight:** Both periods are the same because the force is the same whether the springs are stretched or compressed.
39. **Picture the Problem:** The picture shows the unstretched spring and the spring with a 0.50-kg mass attached to it.

**Strategy:** We can use the displacement of the spring to calculate the spring force constant. The spring force constant and period can then be inserted into the period equation to solve for the necessary mass.

**Solution:** 1. Use the spring force equation to solve for the spring force constant:

\[ F = ky \Rightarrow k = \frac{F}{y} = \frac{mg}{y} \]

2. Insert the numeric values to obtain \( k \):

\[ k = \frac{0.50 \text{ kg} \left(9.81 \text{ m/s}^2\right)}{15 \times 10^{-2} \text{ m}} = 32.7 \text{ N/m} \]

3. Solve the period equation for the mass:

\[ T = 2\pi \sqrt{\frac{m}{k}} \Rightarrow m = \left(\frac{T}{2\pi}\right)^2 k \]

4. Insert the numeric values to obtain the mass:

\[ m = \left(\frac{0.75 \text{ s}}{2\pi}\right)^2 \left(32.7 \text{ N/m}\right) = 0.47 \text{ kg} \]

**Insight:** Since the period is proportional to the square root of the mass, increasing the mass will increase the period.

40. **Picture the Problem:** A mass is attached to a spring and pulled 3.1 cm away from the spring’s equilibrium position and released. The oscillation period and speed are regulated by the stiffness of the spring.

**Strategy:** We can use the spring force constant and the mass to determine the angular frequency \( \omega \). We can combine \( \omega \) with the amplitude to find the maximum speed, and then determine the period directly from \( \omega \).

**Solution:** 1. (a) Use equation 13-10 to find \( \omega \):

\[ \omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{69 \text{ N/m}}{0.57 \text{ kg}}} = 11 \text{ rad/s} \]

3. (b) Multiply the amplitude and angular speed to solve for the maximum velocity:

\[ v_{\text{max}} = A\omega = (0.031 \text{ m})(11 \text{ rad/s}) = 0.34 \text{ m/s} \]

4. (c) Use equation 13-11 to find \( T \):

\[ T = 2\pi \sqrt{\frac{m}{k}} = 2\pi \sqrt{\frac{0.57 \text{ kg}}{69 \text{ N/m}}} = 0.57 \text{ s} \]

**Insight:** The period and frequency are independent of the amplitude. However, the maximum speed does depend upon the amplitude of oscillation.

41. **Picture the Problem:** When the two people enter the car they compress the springs. The distance that the springs are compressed is regulated by their mass and the stiffness of the spring. When the car hits a bump in the road the car begins to oscillate up and down at a frequency determined by the total mass of the car and riders and the stiffness of the springs.

**Strategy:** Using the mass of the two people and the amount the springs compressed, we can calculate the spring force constant. The total load can be obtained by solving the period of oscillation equation for the mass. The mass of the car is found by subtracting the mass of the two people from the total mass.

**Solution:** 1. Solve the force equation for the spring force constant:

\[ F = ky \Rightarrow k = \frac{F}{y} = \frac{mg}{y} \]

2. Enter numeric values for the spring force constant:

\[ k = \frac{125 \text{ kg} \left(9.81 \text{ m/s}^2\right)}{0.0800 \text{ m}} = 1.53 \times 10^4 \text{ N/m} \]

3. (a) Solve the period equation for the total load \( (M+m) \):

\[ T = 2\pi \sqrt{\frac{M+m}{k}} \Rightarrow M + m = \left(\frac{T}{2\pi}\right)^2 k \]
4. Enter numeric values for the total mass:

\[ M + m = \left( \frac{1.65 \text{ s}}{2\pi} \right)^2 (1.53 \times 10^4 \text{ N/m}) = 1060 \text{ kg} \]

5. (b) Subtract the mass of the two people to obtain \( M \):

\[ M = (M + m) - m = 1057 \text{ kg} - 125 \text{ kg} = 932 \text{ kg} \]

Insight: The period of oscillation is an excellent method of determining the mass of an object. This is especially useful in orbit, where conventional scales do not work.

42. Picture the Problem: A mass that is attached to a vertical spring, pulled slightly down from the equilibrium position, and released will oscillate in simple harmonic motion. The acceleration of the mass will be a maximum when the spring is at maximum displacement. As the mass moves back to the equilibrium position the speed increases and the acceleration decreases. The maximum speed is at equilibrium position. As the mass moves away from equilibrium the velocity decreases as the deceleration increases until the mass stops at the opposite amplitude.

Strategy: The period can be found from the spring force constant and mass. From the maximum speed and the period we can calculate the amplitude. We can calculate the maximum acceleration from the period and the amplitude.

Solution: 1. (a) Insert the mass and spring constant into the period equation:

\[ T = 2\pi \sqrt{\frac{m}{k}} = 2\pi \sqrt{\frac{0.85 \text{ kg}}{150 \text{ N/m}}} = 0.47 \text{ s} \]

2. (b) Solve the maximum velocity equation for the amplitude:

\[ A = \frac{v_{\text{max}}}{\omega} = \frac{T v_{\text{max}}}{2\pi} = \frac{0.4730 \text{ s} (0.35 \text{ m/s})}{2\pi} = 2.6 \text{ cm} \]

3. (c) Insert the amplitude and period into the maximum acceleration equation:

\[ a_{\text{max}} = A \omega^2 = A \left( \frac{2\pi}{T} \right)^2 = 0.02635 \text{ m} \left( \frac{2\pi}{0.4730 \text{ s}} \right)^2 = 4.6 \text{ m/s}^2 \]

Insight: Another way to solve this problem is to calculate the angular speed \( \omega = \sqrt{k/m} \) instead of the period. Then \( A = v_{\text{max}} / \omega \) and \( a_{\text{max}} = v_{\text{max}} \omega \) (equations 13-7 and 13-9).

43. Picture the Problem: A mass attached to a spring, displaced slightly from equilibrium, and released will oscillate with simple harmonic motion about the equilibrium position. The period of oscillation is determined by the stiffness of the spring and the mass.

Strategy: Calculate the period of oscillation by dividing the total time \( t \) by the number of oscillations \( n \). Hooke’s Law (equation 6-5) provides a relationship between the mass, spring force constant, and stretch distance. Combine this equation with equation 13-11 to find the stretch distance as a function of the known variables.

Solution: 1. Solve equation 6-5 for \( d \):

\[ F = kd \quad \Rightarrow \quad d = \frac{F}{k} = \frac{m g}{k} = \left( \frac{m}{k} \right) g \]

2. Solve equation 13-11 for \( m/k \):

\[ T = 2\pi \sqrt{\frac{m}{k}} \quad \Rightarrow \quad \frac{m}{k} = \left( \frac{T}{2\pi} \right)^2 \]

3. Insert the expression from step 2 into step 1:

\[ d = \left( \frac{m}{k} \right) g = \left( \frac{T}{2\pi} \right)^2 g \]

4. Calculate the period:

\[ T = \frac{56.7 \text{ s}}{102 \text{ oscillations}} = 0.556 \text{ s} \]

5. Insert the period into the distance equation:

\[ d = \left( \frac{T}{2\pi} \right)^2 g = \left( \frac{0.556 \text{ s}}{2\pi} \right)^2 (9.81 \text{ m/s}^2) = 0.0768 \text{ m} = 7.68 \text{ cm} \]

Insight: The stretch distance can be written as a function of the period. This means that any vertical spring and mass combination that has the same initial stretch distance will oscillate with the same period and frequency.
44. **Picture the Problem:** If the motorcycle is pushed down slightly on its springs it will oscillate up and down in harmonic motion. A rider sitting on the motorcycle effectively increases the mass of the motorcycle and oscillates also.

**Strategy:** We can use the equation for the period of a mass on a spring. Writing this equation for the motorcycle without rider and again for the motorcycle with rider we can calculate the percent difference in the periods.

**Solution:** 1. (a) The period **increases** because the person’s mass is added to the system and \( T \propto \sqrt{m} \).

2. (b) Write the equation for the period of the motorcycle without the rider:
   \[ T = 2\pi \sqrt{\frac{m}{k}} \]

3. Write the equation for the period of the motorcycle with the rider:
   \[ T_2 = 2\pi \sqrt{\frac{m + M}{k}} \]

4. Calculate the percent difference between the two periods:
   \[ \frac{T_2 - T}{T} = \frac{2\pi \sqrt{\frac{m + M}{k}} - 2\pi \sqrt{\frac{m}{k}}}{2\pi \sqrt{\frac{m}{k}}} = \frac{2\pi \sqrt{\frac{m}{k}}}{2\pi \sqrt{\frac{m}{k}}} - 1 = \frac{511 + 122}{511} - 1 = 0.104 = 10.4\% \]

**Insight:** The percent change in the period does not depend on the spring force constant. It only depends on the fractional increase in mass.

45. **Picture the Problem:** The mass on the left is hung from one spring. The mass on the right is hung from two identical springs.

**Strategy:** When a mass is attached to a single spring it stretches by a distance \( x \). When two identical springs are connected end-to-end and the same mass is attached, each spring will stretch by the same amount and the total stretch will be \( 2x \). Since the force has not changed, but the total stretch is twice as much, the effective spring force constant will be half the single force constant (see Problem 83 in Chapter 7). Use this information together with equation 13-11 to find a relationship between the two periods.

**Solution:** 1. (a) The period is **greater than** the period of a single spring because the effective spring force constant is smaller and \( T \propto \frac{1}{\sqrt{k}} \).

2. (b) Write the combined spring force constant as one-half of the single force constant:
   \[ k' = \frac{k}{2} \]

3. Replace \( k' \) with \( k/2 \) in equation 13-11:
   \[ T' = 2\pi \sqrt{\frac{m}{k'}} = 2\pi \sqrt{\frac{m}{(k/2)}} \]

4. Rearrange the equation and replace the term in parentheses with the original period:
   \[ T' = 2\pi \sqrt{\frac{m}{k}} \frac{2\pi \sqrt{m/k}}{\sqrt{2}} = \sqrt{2}T \]

**Insight:** If three identical springs had been connected end-to-end the resulting spring force constant would be \( k' = k/3 \), giving \( T' = \sqrt{3}T \). In general the period will be proportional to the square-root of the length of the spring.
46. **Picture the Problem:** A force is applied to a spring, causing the spring to stretch by a given distance. The resulting force does work on the spring as it is stretched.

**Strategy:** We are given the spring force constant and the stretch distance. These can be directly substituted into the spring work equation (equation 8-4) to obtain the work.

**Solution:** Apply equation 8-4 directly:

\[ W = \frac{1}{2} k x^2 = \frac{1}{2} (9.17 \text{ N/m}) (0.133 \text{ m})^2 = 0.0811 \text{ J} = 81.1 \text{ mJ} \]

**Insight:** The same amount of work is necessary to either stretch or compress the spring by the same distance.

47. **Picture the Problem:** A mass is attached to a spring, displaced from equilibrium, and released from rest. The mass then oscillates about the equilibrium position, converting its energy between spring potential energy and kinetic energy.

**Strategy:** We can use conservation of energy (equations 8-7 and 13-13) to solve this problem because no non-conservative forces act on the spring after it is released. The total energy will be equal to the initial spring potential energy. Find the kinetic energy and the speed when the mass is 0.128 m from equilibrium.

**Solution:**

1. Set \( E_i = E_f \) and substitute expressions for \( K, E, \) and \( U \):

\[ 0 + \frac{1}{2} k A^2 = \frac{1}{2} m v^2 + \frac{1}{2} k x^2 \Rightarrow m v^2 = k \left( A^2 - x^2 \right) \]

2. Solve for the speed:

\[ v = \sqrt{\frac{k}{m}} \left( A^2 - x^2 \right) = \sqrt{\frac{(13.3 \text{ N/m})[0.256^2 \text{ m}^2 - 0.128^2 \text{ m}^2]}{0.321 \text{ kg}}} = 1.43 \text{ m/s} \]

**Insight:** When the mass is located at 0.128 meters it is one-half the maximum distance from equilibrium. At this position one-fourth of the energy is potential energy and three-fourths is kinetic energy.

48. **Picture the Problem:** A mass is attached to a spring, displaced from equilibrium, and released from rest. The mass then accelerates back toward the equilibrium position. The work done to initially stretch the spring is stored as potential energy. This energy is then converted to kinetic energy of the mass as it moves toward the equilibrium position.

**Strategy:** The total mechanical energy equals the initial potential energy of the stretched spring.

**Solution:** Insert the spring force constant and amplitude into the energy equation:

\[ E = \frac{1}{2} k A^2 = \frac{1}{2} (12.3 \text{ N/m}) (0.256 \text{ m})^2 = 0.403 \text{ J} \]

**Insight:** When the spring is released, the potential energy stored in the spring will transfer from potential to kinetic and back again. However, the total energy will remain constant at 0.403 J.

49. **Picture the Problem:** A mass attached to a spring is stretched from equilibrium position. The work done in stretching the spring is stored as potential energy in the spring until the mass is released. After the mass is released, the mass will accelerate, converting the potential energy into kinetic energy. The energy will then transfer back and forth between potential and kinetic energies as the mass oscillates about the equilibrium position.

**Strategy:** The total mechanical energy can be written in terms of the amplitude and spring force constant. The amplitude is given. We can find the spring force constant in terms of the mass and frequency.

**Solution:**

1. Solve the reciprocal of the period equation for the spring force constant:

\[ f = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \Rightarrow k = (2\pi f)^2 m \]

2. Write the energy equation in terms of frequency, mass, and amplitude:

\[ E = \frac{1}{2} k A^2 = \frac{1}{2} \left( 4\pi^2 f^2 m \right) A^2 = 2\pi^2 f^2 m A^2 \]

3. Insert the numeric values to solve for energy:

\[ E = 2\pi^2 \left( 2.6 \text{ Hz} \right)^2 (1.8 \text{ kg}) (0.071 \text{ m})^2 = 1.2 \text{ J} \]

**Insight:** An alternative equation for the energy can be written in terms of the angular speed, mass, and amplitude:

\[ E = \frac{1}{2} m \omega^2 A^2 \]. If you begin with this equation you will obtain the same solution.
50. **Picture the Problem:** A mass attached to a spring is stretched from equilibrium position.

**Strategy:** The work done in stretching the spring is stored as potential energy in the spring until the mass is released. After the mass is released, the mass will accelerate, converting the potential energy into kinetic energy. The energy will then transfer back and forth between potential and kinetic energies as the mass oscillates about the equilibrium position. Solve the conservation of mechanical energy equation, \( E = K + U \), for the kinetic energy. Then use the equation for kinetic energy, \( \frac{1}{2}mv^2 \), to solve for the velocity.

**Solution:**
1. Set \( K = E - U \) and substitute expressions for each term:
   \[
   \frac{1}{2}mv^2 = \frac{1}{2}kA^2 - \frac{1}{2}k\left(\frac{1}{2}A\right)^2
   \]
2. Solve for the speed and simplify:
   \[
   v = \sqrt{\frac{k\left[A^2 - \left(\frac{1}{2}A\right)^2\right]}{m}} = \sqrt{\frac{3kA^2}{4m}}
   \]
3. Insert the numeric values:
   \[
   v = \sqrt{\frac{3(26 \text{ N/m})(0.032 \text{ m})^2}{4(0.40 \text{ kg})}} = 0.22 \text{ m/s}
   \]

**Insight:** When the displacement is half the maximum displacement, the speed is not half the maximum speed. In fact, the speed is \( \frac{\sqrt{3}}{2}v_{\text{max}} \), which is greater than half the speed.

---

51. **Picture the Problem:** A mass attached to a spring is stretched from equilibrium position. The work done in stretching the spring is stored as potential energy in the spring until the mass is released. After the mass is released, the mass will accelerate, converting the potential energy into kinetic energy. The energy will then transfer back and forth between potential and kinetic energies as the mass oscillates about the equilibrium position. The maximum kinetic energy (and speed) occurs when the mass is at the equilibrium position.

**Strategy:** The speed will be a maximum when the total energy is kinetic energy. Insert half the maximum speed into the energy equation in order to solve for the displacement.

**Solution:**
1. (a) Set the maximum kinetic energy equal to the maximum potential energy:
   \[
   K_{\text{max}} = U_{\text{max}}
   \]
   \[
   \frac{1}{2}mv_{\text{max}}^2 = \frac{1}{2}kA^2
   \]
2. Solve for maximum speed:
   \[
   v_{\text{max}} = \sqrt{\frac{k}{m}A^2}
   \]
3. Insert the numeric values:
   \[
   v_{\text{max}} = \sqrt{\frac{26 \text{ N/m}}{0.40 \text{ kg}}} \left(0.032 \text{ m}\right)^2 = 0.26 \text{ m/s}
   \]
4. (b) Insert expressions for \( K, U \), and \( E \) into equation 13-13:
   \[
   K + U = E
   \]
   \[
   \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2
   \]
5. Solve for displacement and replace \( v \) with \( v_{\text{max}}/2 \):
   \[
   x = \sqrt{A^2 - \frac{mv^2}{k}} = \sqrt{A^2 - \frac{m\left(\frac{1}{2}v_{\text{max}}\right)^2}{k}}
   \]
6. Insert the numeric values:
   \[
   x = \sqrt{(0.032 \text{ m})^2 - \frac{(0.40 \text{ kg})(0.129 \text{ m/s})^2}{26 \text{ N/m}}} = 2.8 \text{ cm}
   \]

**Insight:** The mass moves at half its maximum speed at a position greater than half the amplitude.
52. **Picture the Problem**: When the grapes are placed on the scale they cause the spring to stretch beyond its equilibrium point. The grapes will then oscillate up and down on the spring with a period determined by their mass and the stiffness of the spring in the scale.

**Strategy**: Solve the period equation for the unknown mass in terms of the period and the spring force constant. The weight is the mass times the acceleration of gravity.

**Solution**: 1. (a) Solve equation 13-11 for the mass:

\[ T = 2\pi \sqrt{\frac{m}{k}} \Rightarrow m = \left( \frac{T}{2\pi} \right)^2 k \]

2. Insert the numeric values:

\[ m = \left( \frac{0.48 \text{ s}}{2\pi} \right)^2 \left( 650 \text{ N/m} \right) = 3.8 \text{ kg} \]

3. (b) Multiply the mass by \( g \) to find \( W \):

\[ W = mg = (3.8 \text{ kg})(9.81 \text{ m/s}^2) = 37 \text{ N} \]

**Insight**: The period of oscillation could be used to check the accuracy of the scale reading.

53. **Picture the Problem**: When the grapes are placed on the scale they cause the spring to stretch beyond its equilibrium point. The grapes will then oscillate up and down on the spring, and the maximum speed of their oscillation occurs as the spring passes through its equilibrium position.

**Strategy**: Use conservation of energy to equate the maximum potential energy to the maximum kinetic energy and solve for the maximum speed. Eliminate the mass from the equation by using equation 13-11.

**Solution**: 1. Equate the maximum kinetic and potential energies:

\[ K_{\text{max}} = U_{\text{max}} \]

\[ \frac{1}{2}mv_{\text{max}}^2 = \frac{1}{2}kA^2 \]

2. Solve for maximum velocity:

\[ v_{\text{max}} = A \sqrt{\frac{k}{m}} \]

3. Solve the period equation for \( \sqrt{\frac{k}{m}} \):

\[ T = 2\pi \sqrt{\frac{m}{k}} \Rightarrow \sqrt{\frac{k}{m}} = \frac{2\pi}{T} \]

4. Insert that result into the \( v_{\text{max}} \) equation:

\[ v_{\text{max}} = A \sqrt{\frac{k}{m}} = A \left( \frac{2\pi}{T} \right) \]

5. Insert the numeric values:

\[ v_{\text{max}} = (0.023 \text{ m}) \left( \frac{2\pi}{0.48 \text{ s}} \right) = 0.30 \text{ m/s} \]

**Insight**: The maximum speed can be written in terms of the amplitude and period of oscillation. Therefore the spring force constant and mass do not need to be known in order to solve this problem.

54. **Picture the Problem**: A block has kinetic energy as it slides on a frictionless horizontal surface. It encounters an unstretched spring and compresses it before coming to rest.

**Strategy**: When the mass first encounters the spring the energy is all kinetic. As the spring is compressed the energy is converted to spring potential energy. Equate the energies in order to solve for the spring force constant. The motion corresponds to one-fourth of a period. A stiffer spring will cause the mass to stop in a shorter distance, and therefore a shorter time period.

**Solution**: 1. (a) Equate the kinetic and potential energies:

\[ K_i = U_f \Rightarrow \frac{1}{2}mv_i^2 = \frac{1}{2}kA^2 \]

2. Solve for the spring force constant:

\[ k = m \left( \frac{v_i}{A} \right)^2 \]

3. Insert the numeric values:

\[ k = 0.505 \text{ kg} \left( \frac{1.18 \text{ m/s}}{0.232 \text{ m}} \right)^2 = 13.1 \text{ N/m} \]
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4. (b) Solve for one-quarter period:

\[
\frac{1}{4} T = \frac{\pi}{2} \sqrt{\frac{m}{k}} = \frac{\pi}{2} \sqrt{\frac{0.505 \text{ kg}}{13.06 \text{ N/m}}} = 0.309 \text{ s}
\]

5. (c) When the force constant increases the time to stop decreases. A greater force constant means a stiffer spring and a greater stopping force, therefore a shorter stopping time.

**Insight:** After the spring has stopped the block all of the energy is in the compressed spring. However, the spring will push back on the block, accelerating it back the way it came. When the block leaves the spring it will have the same speed, but travel in the opposite direction as when it first encountered the spring. The accelerating time is equal to the stopping time.

### Picture the Problem

45. **Picture the Problem:** A block and spring are initially at rest as a bullet is fired at high speed directly toward them. The bullet then embeds in the block and compresses the spring.

**Strategy:** The bullet and block first undergo an inelastic collision. Then they jointly compress the spring, converting their kinetic energy into potential energy of the spring. Use conservation of energy to relate the speed \(v\) of the block and bullet to the compression distance \(x\). Finally, use conservation of momentum to find the initial speed of the bullet \(v_0\) from the combined speed of bullet and block. The time elapsed from impact to rest is one-quarter of a period.

**Solution:**

1. (a) Set the initial kinetic energy of the block and bullet to the final potential energy of the spring:

\[
\frac{1}{2} (M + m) v^2 = \frac{1}{2} k A^2
\]

2. Solve for the speed of the bullet and block:

\[
v = \sqrt{\frac{k A^2}{M + m}} = \sqrt{\frac{785 \text{ N/m} \cdot 0.0588 \text{ m}^2}{1.500 \text{ kg} + 0.00225 \text{ kg}}} = 1.344 \text{ m/s}
\]

3. Using conservation of momentum write the initial speed of the bullet in terms of the final speed of bullet and block:

\[
m v_0 = (M + m) v
\]

\[
v_0 = \left(\frac{M + m}{m}\right) v = \left(\frac{1.500 \text{ kg} + 0.00225 \text{ kg}}{0.00225 \text{ kg}}\right) (1.344 \text{ m/s}) = 897 \text{ m/s}
\]

4. Calculate initial speed of bullet:

\[
v_0 = \frac{1.500 \text{ kg} + 0.00225 \text{ kg}}{0.00225 \text{ kg}} \cdot 1.344 \text{ m/s} = 897 \text{ m/s}
\]

5. (b) Calculate one-quarter period:

\[
\frac{T}{4} = \frac{\pi}{2} \sqrt{\frac{M + m}{k}} = \frac{\pi}{2} \sqrt{\frac{1.50225 \text{ kg}}{785 \text{ N/m}}} = 0.0687 \text{ s}
\]

**Insight:** The initial kinetic energy of the bullet does not equal the final energy of the compressed spring. Some of the initial kinetic energy is lost due to the inelastic collision with the block.

### Picture the Problem

46. **Picture the Problem:** The picture shows a metronome that oscillates with a mass (the bow tie) attached to a thin metal rod that pivots about a point near the belly of the penguin.

**Strategy:** The device can be considered a physical pendulum whose moment of inertia about the pivot point can be adjusted by moving the bow tie up and down the thin metal rod. Use equation 13-21 to answer the conceptual question.

**Solution:** Equation 13-21 indicates that the period of a physical pendulum is proportional to the square root of the moment of inertia. In order to reduce the period and increase the frequency of oscillation the moment of inertia should be decreased. We conclude that the penguin’s bow tie should be moved downward in order to increase the frequency.

**Insight:** The reverse would be true if this were a standard pendulum with the pivot point at the top and the mass down below. In such a case the mass would have to be moved upward in order to decrease the moment of inertia.
57. **Picture the Problem:** A grandfather clock keeps correct time at sea level but is taken to the top of a nearby mountain.

**Strategy:** The acceleration of gravity will be slightly smaller at high altitude than it is at sea level because the mountaintop location is farther from the center of the Earth. Use this fact together with equation 13-20 to answer the conceptual question.

**Solution:** (a) Equation 13-20 indicates that the period of a pendulum is inversely proportional to the square root of the acceleration of gravity. Decreasing $g$ will therefore increase $T$. We conclude that if the clock is taken to the top of a nearby mountain it will run slow.

2. (b) The best explanation is **I.** Gravity is weaker at the top of the mountain, leading to a greater period of oscillation. Statement II is partly true but ignores the change in the acceleration of gravity. Statement III could only be true if the mountain were filled with material with a higher than average density, so that the nearby location of extra mass more than compensates for the increased distance from the center of the Earth.

**Insight:** Some gravity meters operate on this principle, precisely determining the local acceleration of gravity by accurately measuring the period of a pendulum.

58. **Picture the Problem:** A pendulum of length $L$ oscillates with a period $T$.

**Strategy:** Consider the relationship between the period and the length of a pendulum (equation 13-20) in order to answer the conceptual question.

**Solution:** Equation 13-20 indicates that the period of a pendulum is directly proportional to the square root of the length. In order to double the period you must therefore quadruple the length. We conclude that the length of the pendulum must be increased to $4L$.

**Insight:** The pendulum of a grandfather clock that oscillates with a period of 1.00 s must therefore be four times longer than a similar pendulum for a smaller clock that is designed to oscillate with a period of 0.50 s.

59. **Picture the Problem:** The microphone behaves as a simple pendulum.

**Strategy:** We can use the period of the pendulum to determine its length.

**Solution:** 1. Solve equation 13-20 for $L$:

$$ T = 2\pi \sqrt{\frac{L}{g}} \Rightarrow L = \left( \frac{T}{2\pi} \right)^2 g $$

2. Insert the numeric values:

$$ L = \left( \frac{1}{\pi} \cdot 60.0 \text{ s} \right)^2 \left( 9.81 \text{ m/s}^2 \right) = 8.95 \text{ m} $$

**Insight:** A long pendulum will oscillate slowly. This problem shows that it takes 6 seconds for one oscillation of a pendulum about 9 meters long. A shorter pendulum, such as that on a grandfather clock, is only about a meter long and has a period of about 2 seconds.

60. **Picture the Problem:** A mass is attached to the end of a 2.5-meter-long string, displaced slightly from the vertical and released. The mass then swings back and forth through the vertical with a period determined by the length of the string.

**Strategy:** Use the period of the pendulum and its length to calculate the acceleration of gravity.

**Solution:** 1. Solve the period equation for gravity:

$$ T = 2\pi \sqrt{\frac{L}{g}} \Rightarrow g = \left( \frac{2\pi}{T} \right)^2 L $$

2. Insert the numeric values:

$$ g = \left( \frac{2\pi}{\frac{1}{2} (16 \text{ s})} \right)^2 (2.5 \text{ m}) = 9.6 \text{ m/s}^2 $$

**Insight:** The small variations in gravity around the surface of the Earth are measured using the period of a pendulum.
61. **Picture the Problem**: The pendulum mass is displaced slightly from equilibrium and oscillates back and forth through the vertical.

**Strategy**: The time the pendulum takes to move from maximum displacement to equilibrium position is one-quarter of a period. Use equation 13-20 to determine the time.

**Solution**: Insert the numeric values into equation 13-20 and convert feet to meters:

\[
T = \frac{\pi}{4} \sqrt{\frac{L}{g}} = \frac{\pi}{4} \sqrt{\frac{75.0 \text{ ft}}{9.81 \text{ m/s}^2}} \left(\frac{0.305 \text{ m}}{\text{ft}}\right) = 2.4 \text{ s}
\]

**Insight**: The full period of this pendulum is 4(2.4 s) = 9.6 seconds. A pendulum with only half this length would have a period of 6.8 s.

62. **Picture the Problem**: A simple pendulum is a mass attached to a string. The mass is displaced so the string is slightly away from the vertical and released. The mass then oscillates about the vertical with a period determined by the length of the string and gravity.

**Strategy**: Calculate the length of the pendulum from its period.

**Solution**: 1. Solve the period equation for length:

\[
T = 2\pi \sqrt{\frac{L}{g}} \Rightarrow L = \left(\frac{T}{2\pi}\right)^2 g
\]

2. Insert the numeric values:

\[
L = \left(\frac{1.00 \text{ s}}{2\pi}\right)^2 (9.81 \text{ m/s}^2) = 24.8 \text{ cm}
\]

**Insight**: This is the length of the pendulum in many older clocks. Larger clocks, such as a grandfather clock, have pendulums about a meter long with a period of 2 seconds.

63. **Picture the Problem**: The pendulum on the Moon is the same length string and mass, with the mass displaced from the vertical and released. The period is determined by the length of the string and the acceleration due to gravity.

**Strategy**: The period of the pendulum on the Moon can be calculated by replacing the acceleration of gravity on the Earth with the acceleration of gravity on the Moon, \(g_{\text{Moon}} = \frac{1}{6} g_{\text{Earth}}\), in equation 13-20.

**Solution**: 1. (a) The period of a pendulum is inversely proportional to the square-root of the acceleration of gravity. Therefore, if the pendulum is taken to the Moon, where gravity is weaker, its period would increase.

2. (b) Write the period on the Moon in terms of the period on Earth:

\[
T_{\text{Moon}} = 2\pi \sqrt{\frac{L}{\frac{1}{6} g_{\text{Earth}}}} = \sqrt{6} \cdot 2\pi \sqrt{\frac{L}{g_{\text{Earth}}}} = \sqrt{6} \cdot T_{\text{Earth}}
\]

2. Calculate the period on the Moon:

\[
T_{\text{Moon}} = \sqrt{6} (1.00 \text{ s}) = 2.45 \text{ s}
\]

**Insight**: A grandfather clock taken to the Moon would run 2.45 times slower than one on the Earth. To run properly, the pendulum in the clock would need to be shortened to one-sixth of its original length.

64. **Picture the Problem**: A hula-hoop rocks back and forth while suspended from a peg.

**Strategy**: Calculate the period of the hula-hoop’s oscillation by treating it as a physical pendulum (equation 13-21). The length of the pendulum (distance of the pivot to the center of mass) is the radius of the hula-hoop. The moment of inertia is given in the problem.

**Solution**: 1. Insert \(l = R\) and \(I = 2mR^2\) into equation 13-21:

\[
T = 2\pi \sqrt{\frac{l}{g}} \frac{I}{m} = 2\pi \sqrt{\frac{R}{g}} \frac{2mR^2}{mR^2} = 2\pi \sqrt{\frac{2R}{g}}
\]

2. Simplify the equation:

\[
T = 2\sqrt{2\pi} \sqrt{\frac{R}{g}}
\]

**Insight**: The final equation could be rewritten as \(T = 2\pi \sqrt{2R/g} = 2\pi \sqrt{D/g}\), which is the same as the period of the simple pendulum with length equal to the diameter of the hoop. This result is unique to the hoop and does not apply to all physical pendulums.
65. **Picture the Problem**: The hat rocks back and forth on the peg.

**Strategy**: Treat the hat as a physical pendulum and solve for the moment of inertia from the period equation.

**Solution**: 1. Solve equation 13-21 for the moment of inertia:

\[
T = 2\pi \sqrt{\frac{L}{g}} \sqrt{\frac{I}{mL^2}} \Rightarrow I = \frac{mgL^2}{4\pi^2}
\]

2. Solve for the moment of inertia:

\[
I = \frac{(0.98 \text{ kg})(9.81 \text{ m/s}^2)(0.084 \text{ m})(0.73 \text{ s})^2}{4\pi^2} = 0.011 \text{ kg} \cdot \text{m}^2
\]

**Insight**: Measuring the period of oscillation of an irregularly shaped object is a quick and simple way of determining the object’s moment of inertia.

66. **Picture the Problem**: A meter stick oscillates about one end.

**Strategy**: The period of the meter stick can be found by treating the meter stick as a physical pendulum. For a meter stick pivoted at one end, the center of mass is \(\ell = L/2\) and the moment of inertia is \(I = \frac{1}{3} mL^2\).

**Solution**: 1. (a) The period of a simple pendulum of length one meter will be greater than that of the meter stick because the center of mass of the meter stick is only 50 cm from the pivot point, so its effective length is shorter than a one-meter simple pendulum.

2. (b) Insert the length and moment of inertia of the meter stick into the physical pendulum equation:

\[
T_{\text{meter stick}} = 2\pi \sqrt{\frac{L}{g}} \sqrt{\frac{\frac{1}{3} mL^2}{m(\frac{1}{2}L)^2}} = 2\pi \sqrt{\frac{\frac{1}{3}L}{g}}
\]

3. Set the period of the simple pendulum equal to the period of the meter stick:

\[
2\pi \sqrt{\frac{\ell}{g}} = 2\pi \sqrt{\frac{\frac{1}{3}L}{g}}
\]

4. Solve for the length of the simple pendulum:

\[
\ell = \frac{2}{3} L = 0.667 \text{ m in length}
\]

**Insight**: The period of the meter stick is less than the period of a simple pendulum with length equal to one meter. The period is larger than a simple pendulum with length equal to the distance from the center of mass to the pivot.

67. **Picture the Problem**: A steel beam oscillates about one end.

**Strategy**: Use the equation for the period of a physical pendulum to solve for the length of the beam. For a uniform beam suspended from one end, \(\ell = L/2\) and \(I = \frac{1}{3} mL^2\).

**Solution**: 1. Substitute for \(\ell\) and \(I\) in equation 13-21:

\[
T = 2\pi \sqrt{\frac{\frac{1}{3}L}{g}} \sqrt{\frac{\frac{1}{3} mL^2}{m(\frac{1}{2}L)^2}} = 2\pi \sqrt{\frac{\frac{1}{3}L}{g}}
\]

2. Solve for the length of the beam:

\[
L = \frac{3}{2} \left(\frac{T}{2\pi}\right)^2 g = \frac{3}{2} \left(\frac{2.00 \text{ s}}{2\pi}\right)^2 (9.81 \text{ m/s}^2) = 1.49 \text{ m}
\]

**Insight**: This physical pendulum is longer than the 0.994 m length of a simple pendulum that has a period of 2.00 s.
68. **Picture the Problem:** As a child walks, each leg swings like a pendulum with an amplitude of about one radian.

**Strategy:** The leg can be treated as a physical pendulum, with center of mass, \( \ell = L/2 \), and moment of inertia, \( I = \frac{1}{3} mL^2 \). The period is found directly from the physical pendulum equation. We can assume that the leg swings through a distance of about 1.0 radian per step. The walking speed is then the arc length per step divided by half the period. We use half the period as walking involves two legs.

**Solution:** 1. Insert the center of mass length and moment of inertia into the equation for the physical pendulum:

\[
T = 2\pi \sqrt{\frac{L}{g}} \left[ \sqrt{\frac{\frac{1}{3} mL^2}{m\left(\frac{1}{2} L\right)^2}} \right] = 2\pi \sqrt{\frac{2L}{3g}} = 2\pi \sqrt{\frac{2(0.55 \text{ m})}{3(9.81 \text{ m/s}^2)}} = 1.2 \text{ s}
\]

2. (b) Calculate the distance traveled per step:

\[
d = L\theta = 0.55 \text{ m}
\]

3. Divide the distance by half a period to calculate walking speed:

\[
v = \frac{d}{\frac{T}{2}} = \frac{0.55 \text{ m}}{0.607 \text{ s}} = 0.91 \text{ m/s}
\]

**Insight:** The distance per step is proportional to the leg length. The period is proportional to the square root of the leg length. Combining these together shows that a person’s walking speed is proportional to the square root of his/her leg length.

69. **Picture the Problem:** A pendulum is made by attaching a mass to the end of a string inside an elevator. The pendulum oscillates back and forth with a period determined by the length of the string and the effective acceleration of gravity experienced in the elevator.

**Strategy:** The motion of the pendulum is determined by its length and the effective acceleration of gravity in the elevator. As the elevator accelerates upward the effective acceleration of gravity is \( g + a \). When the elevator accelerates downward the effective acceleration of gravity is \( g - a \). To find the period of the pendulum in the elevator we can substitute the effective acceleration of gravity into the period equation.

**Solution:** 1. (a) Replace \( g \) by \( g + a \) in the equation for the period of a pendulum:

\[
T = 2\pi \sqrt{\frac{L}{g + a}}
\]

2. (b) Replace \( g \) by \( g - a \) in the equation for the period of a pendulum:

\[
T = 2\pi \sqrt{\frac{L}{g - a}}
\]

**Insight:** Consider the effect on the pendulum if the elevator were to be in free fall. According to our answer to part (b), as the downward acceleration approaches \( g \), the period increases. In the limit that \( a \rightarrow g \), \( T \rightarrow \infty \). If the elevator were in free fall the tension in the pendulum string would be zero and the pendulum would not oscillate.

70. **Picture the Problem:** An object undergoes simple harmonic motion with a period \( T \). In the time \( 3T/2 \) the object moves through a total distance of \( 12D \).

**Strategy:** Use the principles of simple harmonic motion to answer the conceptual question.

**Solution:** An object that undergoes simple harmonic motion moves through a distance of \( 4A \) during each cycle, where \( A \) is the amplitude of the motion. It therefore travels a distance \( 6A \) in the time \( 3T/2 \). We conclude that the distance \( 6A = 12D \) and that \( A = 2D \).

**Insight:** Another approach is to note that the object travels a distance \( \frac{12D}{3/2} = 8D \) in one period of oscillation. We can then set \( 8D = 4A \) and find that \( A = 2D \).
71. **Picture the Problem**: A mass on a spring oscillates with a period $T$, which is later doubled without changing either the amplitude or the spring force constant.
   **Strategy**: Use the principles of simple harmonic motion for a mass on a spring to answer the conceptual questions.
   **Solution**: 1. (a) Doubling the period will cut the frequency in half. Therefore the angular frequency is changed by a multiplicative factor of $\frac{1}{2}$.
   2. (b) Doubling the period will change the frequency by a multiplicative factor of $\frac{1}{2}$.
   3. (c) The maximum speed $v_{\text{max}} = A\omega$ is linearly proportional to the angular frequency, so cutting the frequency in half while keeping the amplitude constant will also change the maximum speed by a multiplicative factor of $\frac{1}{2}$.
   4. (d) The maximum acceleration is given by $a_{\text{max}} = A\omega^2$, so cutting the frequency in half while keeping the amplitude constant will change the maximum acceleration by a multiplicative factor of $\left(\frac{1}{2}\right)^2 = \frac{1}{4}$.
   5. (e) The total mechanical energy is given by $E = \frac{1}{2}kx^2$ (equation 13-15). If neither the spring force constant $k$ nor the amplitude $A$ is changed, the total energy must remain constant, or it is changed by a multiplicative constant $1.00$.
   **Insight**: At first glance it would seem as if the total mechanical energy must decrease because $v_{\text{max}}$ has been cut in half, implying $E = \frac{1}{2}mv_{\text{max}}^2$ has decreased. However, in order to double the period $T = 2\pi\sqrt{\frac{m}{k}}$ without changing the force constant, the mass must be quadrupled. Therefore, $K_{\text{max}} = \frac{1}{2}(4m)(v_{\text{max}}/2)^2 = \frac{1}{2}mv_{\text{max}}^2$ and the total energy remains exactly the same.

72. **Picture the Problem**: An object undergoes simple harmonic motion with amplitude $A$, which is later doubled.
   **Strategy**: Use the principles of simple harmonic motion to answer the conceptual questions.
   **Solution**: 1. (a) Doubling the amplitude will not change the angular frequency; the multiplicative factor is $1.00$.
   2. (b) Doubling the amplitude will not change the frequency; the multiplicative factor is $1.00$.
   3. (c) Doubling the amplitude will not change the period; the multiplicative factor is $1.00$.
   4. (d) The maximum speed is linearly proportional to the amplitude, so doubling the amplitude will also change the maximum speed by a multiplicative factor of $2.00$.
   5. (e) The maximum acceleration is linearly proportional to the amplitude, so doubling the amplitude will change the maximum acceleration by a multiplicative factor of $2.00$.
   6. (f) The total mechanical energy is proportional to the square of the amplitude, so doubling the amplitude will change the total mechanical energy by a multiplicative factor of $4.00$.
   **Insight**: If this oscillator were a mass on a spring, you must store four times as much spring potential energy $\frac{1}{2}kx^2$ in order to double the maximum stretch distance $x$. It can also be shown (with some difficulty) that you must store four times as much gravitational potential energy in a pendulum if you double the amplitude.

73. **Picture the Problem**: A mass $m$ is suspended from the ceiling of an elevator by a spring of force constant $k$. When the elevator is at rest, the period of its motion is $T$.
   **Strategy**: Use the principles of simple harmonic motion to answer the conceptual question.
   **Solution**: 1. (a) When the elevator moves upward with constant speed there is no detectable difference from when it is at rest and no change in the behavior of the mass and the spring. Therefore, the period will remain the same.
   2. (b) When the elevator moves upward with constant acceleration the system behaves as if the acceleration of gravity were larger. Therefore, the spring is stretched by a greater amount, but from $T = 2\pi\sqrt{\frac{m}{k}}$ (equation 13-11) we see that its period of oscillation will remain the same.
   **Insight**: The acceleration of the elevator creates an effective acceleration of gravity in the elevator. As the elevator accelerates upward the effective acceleration of gravity is $g + a$, and when the elevator accelerates downward it is $g − a$. The period of a mass on a spring, however, is independent of the acceleration of gravity.
74. **Picture the Problem:** A pendulum of length $L$ is suspended from the ceiling of an elevator. When the elevator is at rest, the period of the pendulum is $T$.

**Strategy:** The acceleration of the elevator creates an effective acceleration of gravity in the elevator. As the elevator accelerates upward the effective acceleration of gravity is $g + a$, and when the elevator accelerates downward it is $g - a$. The motion of the pendulum is determined by this effective acceleration of gravity.

**Solution:** 1. (a) When the elevator moves upward with constant speed there is no detectable difference from when it is at rest and no change in the behavior of the pendulum. Therefore, the period will remain the same.

2. (b) When the elevator moves upward with constant acceleration the system behaves as if the acceleration of gravity were larger. Therefore, we can see from $T = 2\pi \sqrt{L/g}$ (equation 13-11) that its period of oscillation will decrease.

**Insight:** If the elevator were to accelerate downward at 9.81 m/s$^2$, as would happen if the cable were cut, the effective acceleration of gravity would be zero inside the elevator and the pendulum would not swing at all.

75. **Picture the Problem:** A mass attached to a spring is displaced from equilibrium and released. The mass oscillates about the spring’s equilibrium position. As the spring is oscillating, the energy transfers back and forth between kinetic energy of the mass (maximum when the mass is at equilibrium) and potential energy of the spring (maximum when the mass is at the maximum displacement from equilibrium).

**Strategy:** Use conservation of energy to solve for the speed when the spring is halfway to equilibrium.

**Solution:** 1. Set the initial total energy equal to the final total energy.

\[ K_i + U_i = K_f + U_f \]

\[ \frac{1}{2} m v_i^2 + \frac{1}{2} k x_i^2 = \frac{1}{2} m v_f^2 + \frac{1}{2} k x_f^2 \]

2. Solve for the final speed:

\[ v_f = \sqrt{v_i^2 + \frac{k}{m} (x_f^2 - x_i^2)} \]

3. Insert the numeric values:

\[ v_f = \sqrt{(0.25 \text{ m/s})^2 + \frac{59 \text{ N/m}}{1.8 \text{ kg}} (0.084 \text{ m}^2 - 0.042 \text{ m}^2)} = 0.49 \text{ m/s} \]

**Insight:** You can calculate that the speed at the equilibrium point is 0.54 m/s. The speed halfway to equilibrium is much closer to the maximum speed than it is to the initial speed.

76. **Picture the Problem:** An astronaut uses a Body Mass Measurement Device (BMMD) while in orbit.

**Strategy:** The BMMD can be treated as a mass on a spring. Use equation 13-11 to solve for the mass.

**Solution:** 1. Solve the period equation for the mass:

\[ T = 2\pi \sqrt{\frac{m}{k}} \Rightarrow m = \frac{kT^2}{4\pi^2} \]

2. Insert the numeric values:

\[ m = \frac{(2600 \text{ N/m})(0.85 \text{ s})^2}{4\pi^2} = 48 \text{ kg} \]

**Insight:** When using a BMMD an astronaut must remain as motionless as possible. Moving arms and legs around, such that they are not moving at the same frequency as the spring, can alter the reading.
77. **Picture the Problem**: Atoms in a solid form a lattice with a fixed distance between each atom. When an atom is displaced slightly from this equilibrium distance, it will oscillate much the same as a mass on a spring.

**Strategy**: Write the maximum acceleration in terms of amplitude and frequency (equation 13-9). Then divide by the acceleration of gravity to find the maximum acceleration in terms of $g$.

**Solution**:
1. Write the acceleration as a function of frequency and amplitude:
   \[ a_{\text{max}} = A\omega^2 \]
   \[ = A(2\pi f)^2 \]
2. Insert the numeric values:
   \[ a_{\text{max}} = (0.10 \times 10^{-10} \text{ m})(2\pi \times 10^{12} \text{ s}^{-1})^2 = 4 \times 10^{14} \text{ m/s}^2 \]
3. Divide by the acceleration of gravity:
   \[ a_{\text{max}} = 3.95 \times 10^{14} \text{ m/s}^2 \left( \frac{g}{9.81 \text{ m/s}^2} \right) = (4 \times 10^{13})g \]

**Insight**: Since this value is so small, the effect of gravity on the individual atoms can be ignored when dealing with interatomic vibrations.

78. **Picture the Problem**: Sunspots are dark spots on the surface of the Sun caused by magnetic fields in the Sun. The density of sunspot varies with time with a regular period. The sunspots are currently at a minimum in their cycle.

**Strategy**: The frequency is the number of cycles divided by the time.

**Solution**: Take the inverse of the period and convert the units to hertz:
   \[ f = \frac{1}{T} = \left( \frac{1}{11 \text{ yr}} \right) \left( \frac{\text{yr}}{3.156 \times 10^7 \text{ s}} \right) = 2.9 \times 10^{-9} \text{ Hz} \]

**Insight**: Frequencies can range from very small (such as the frequency of the sunspot cycle) to very high frequencies such as the frequency of gamma radiation ($10^{25}$ Hz).

79. **Picture the Problem**: The figure shows the cantilever with a 50-nanometer gold dot near its tip.

**Strategy**: Solve the period equation for a mass on a spring (equation 13-11) for the spring force constant.

**Solution**: Solve equation 13-11 for $k$ and rewrite in terms of $f$:
   \[ k = \left( \frac{2\pi}{T} \right)^2 m = (2\pi f)^2 m = \left[ 2\pi (14.5 \times 10^9 \text{ Hz}) \right]^2 \left( 6.65 \times 10^{-16} \text{ kg} \right) = 5.52 \text{ N/m} \]

**Insight**: With the known spring force constant, it is possible to use the cantilever to measure very small masses by measuring the frequency of oscillation.

80. **Picture the Problem**: An object undergoing simple harmonic motion with a period $T$ is at the position $x = 0$ at the time $t = 0$. At the time $t = 0.25T$ the position of the object is positive.

**Strategy**: The graph of the position of the object as a function of time would resemble the one shown at right. Compare the times specified in the problem with the graph to determine whether the position of the object is positive, negative, or zero.

**Solution**:
1. (a) At $t = 1.5T$ the position $x$ of the object is **zero**.
2. (b) At $t = 2.0T$ the position $x$ of the object is **zero**.
3. (c) At $t = 2.25T$ the position $x$ of the object is **positive**.
4. (d) At $t = 6.75T$ (not shown on the graph but similar to $0.75T$) the position $x$ of the object is **negative**.

**Insight**: The expression for this object’s position would be $A\sin(\omega t)$ instead of $A\cos(\omega t)$ because its position is zero at time $t = 0$. 

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81. **Picture the Problem:** A mass that is attached to a spring, displaced from equilibrium, and released, will oscillate with simple harmonic motion. The maximum speed occurs as the mass passes through the equilibrium position. The maximum force occurs when the spring is stretched its maximum distance from equilibrium.

**Strategy:** Find the amplitude from the maximum speed and maximum acceleration. Then determine the maximum acceleration from the maximum force and Newton’s Second Law. Then obtain the angular speed \( \omega \) from the maximum speed and acceleration, and finally calculate the force constant and frequency of oscillation from \( \omega \).

**Solution:** 1. (a) Combine the maximum velocity and maximum acceleration equations to solve for the amplitude:

\[
\begin{align*}
 v_{\text{max}} &= A\omega, \\
 a_{\text{max}} &= A\omega^2
\end{align*}
\]

\[
\begin{align*}
 \frac{v_{\text{max}}^2}{a_{\text{max}}} &= (A\omega)^2 = A
\end{align*}
\]

2. Using Newton’s Second Law write amplitude as a function of the maximum force:

\[
A = \frac{v_{\text{max}}^2}{a_{\text{max}}} = \frac{v_{\text{max}}^2}{F_{\text{max}}/m} = \frac{mv_{\text{max}}^2}{F_{\text{max}}}
\]

3. Solve for the amplitude:

\[
A = \frac{(3.1 \text{ kg})(0.68 \text{ m/s})^2}{11 \text{ N}} = 0.13 \text{ m}
\]

4. (b) Combine maximum velocity and acceleration to find angular speed:

\[
\omega = \frac{a_{\text{max}}}{v_{\text{max}}} = \frac{F_{\text{max}}}{mv_{\text{max}}} = \frac{11 \text{ N}}{(3.1 \text{ kg})(0.68 \text{ m/s})} = 5.2 \text{ rad/s}
\]

5. Solve equation 13-10 for \( k \):

\[
k = m\omega^2 = (3.1 \text{ kg})(5.2 \text{ rad/s})^2 = 84 \text{ N/m}
\]

6. (c) Use the angular speed to determine \( f \):

\[
f = \frac{\omega}{2\pi} = \frac{5.2 \text{ rad/s}}{2\pi} = 0.83 \text{ Hz}
\]

**Insight:** Another way to solve this problem is to first calculate the angular speed from the maximum force and velocity. The amplitude can then be found from the maximum speed divided by the angular speed.

82. **Picture the Problem:** A mass that is attached to a spring, displaced from equilibrium, and released, will oscillate with simple harmonic motion. The acceleration oscillates with maxima when the mass is displaced farthest from equilibrium and minimum acceleration as the mass passes through equilibrium.

**Strategy:** Determine the frequency from the angular speed, and then find the maximum speed and the amplitude from the maximum acceleration and the angular speed.

**Solution:** 1. (a) Divide the angular speed by \( 2\pi \) to find the frequency:

\[
f = \frac{\omega}{2\pi} = \frac{2.41 \text{ rad/s}}{2\pi} = 0.384 \text{ Hz}
\]

2. (b) Combine equations 13-7 and 13-9 to find \( v_{\text{max}} \) in terms of \( a_{\text{max}} \) and \( \omega \):

\[
v_{\text{max}} = A\omega = \left(\frac{a_{\text{max}}}{\omega^2}\right)\omega = \frac{a_{\text{max}}}{\omega} \cdot \frac{0.302 \text{ m/s}^2}{2.41 \text{ rad/s}} = 0.125 \text{ m/s}
\]

3. (c) Solve for the amplitude from the maximum acceleration equation:

\[
A = \frac{a_{\text{max}}}{\omega^2} = \frac{0.302 \text{ m/s}^2}{(2.41 \text{ rad/s})^2} = 5.20 \text{ cm}
\]

**Insight:** It is always possible to calculate the amplitude, maximum velocity, and maximum acceleration, when given the equation for the displacement, velocity, or acceleration.
83. **Picture the Problem**: The Sun vibrates in a complicated three-dimensional pattern but with a known period.  

**Strategy**: Use the period to determine the angular frequency, and then determine the amplitude of oscillation from the maximum speed and angular frequency.  

**Solution**:  
1. (a) Solve for the angular speed:  
   \[ \omega = \frac{2\pi}{T} = \frac{2\pi}{5.7 \text{ min}} \frac{60 \text{ s}}{1 \text{ min}} = 0.018 \text{ rad/s} \]  
   2. (b) Use the angular speed to solve for the amplitude:  
   \[ A = \frac{v_{\text{max}}}{\omega} = \frac{4.5 \text{ m/s}}{0.018 \text{ rad/s}} = 250 \text{ m} \]  

**Insight**: By tracking the oscillation patterns in the Sun, helioseismologists can study the solar dynamics.

84. **Picture the Problem**: The bullet travels toward the pendulum bob, where it undergoes an inelastic collision. The bob and bullet then rise to a height 12.4 cm above their initial position.  

**Strategy**: Use conservation of energy to find the speed of the bullet and bob just after the collision. Then use conservation of momentum to find the speed of the bullet just before the collision. The time is one-quarter of a period.  

**Solution**:  
1. (a) Since the collision between the bullet and bob is inelastic, the kinetic energy immediately after the collision will be less than the kinetic energy before the collision.  
2. (b) Set the kinetic energy after the collision equal to the potential energy at the highest point:  
   \[ \frac{1}{2}(M + m)v^2 = (M + m)gh \]  
3. Solve for the speed of bullet and bob:  
   \[ v = \sqrt{2gh} = \sqrt{2(9.81 \text{ m/s}^2)(0.124 \text{ m})} = 1.56 \text{ m/s} \]  
4. Use conservation of momentum during the collision to solve for \( v_0 \):  
   \[ m\frac{v_0}{m} = (M + m)v \]  
   \[ v_0 = (M + m)v = \frac{1.45 \text{ kg} + 0.00950 \text{ kg}}{0.00950 \text{ kg}} = 1.56 \text{ m/s} = 0.240 \text{ km/s} \]  
5. (c) Use the period equation to solve for one-quarter period:  
   \[ t = \frac{1}{4}T = \frac{1}{4} \left( \frac{L}{g} \right) = \frac{\pi}{2} \sqrt{\frac{0.745 \text{ m}}{9.81 \text{ m/s}^2}} = 0.433 \text{ s} \]  

**Insight**: The final height of the bullet and block is independent of the length of the pendulum string. However, the time required to reach the final height (and come to a rest) does depend on the string length because a longer pendulum will have a longer oscillation period.

85. **Picture the Problem**: This is a damped harmonic motion problem.  

**Strategy**: When the oscillation has damped by 10%, the amplitude is 90% of the original amplitude. We can solve for the time when the amplitude is 90% of the original amplitude.  

**Solution**:  
1. Set the amplitude equal to 0.9 times the initial amplitude and cancel out \( A_0 \):  
   \[ A = 0.900A_0 = A_0 e^{-b/2m} \]  
   \[ 0.900 = e^{-b/2m} \]  
2. Take natural log of both sides and solve for \( t \):  
   \[ t = -\frac{2m}{b}(\ln 0.900) = -\frac{2(0.00144 \text{ kg})}{3.30 \times 10^{-5} \text{ kg/s}}(\ln 0.900) = 9.20 \text{ s} \]  

**Insight**: A common mistake in damping problems is misinterpreting the decrease in amplitude. The amplitude \( A \) is the remaining amplitude, not the decrease in the amplitude. When the amplitude has decreased by a fractional amount \( x \), the remaining amplitude is always \( A = A_0(1 - x) \).
86. **Picture the Problem:** The figure shows a cosine wave of amplitude $A$ and period $T$. The wave decreases from a displacement of $A$ to $A/2$ over the time period $t$.

**Strategy:** For a cosine wave the initial displacement is $A$. Determine the time $t$ it takes for the cosine function to decrease from $A$ to $A/2$.

**Solution:** 1. Set the displacement equation equal to $A/2$ and simplify:
\[
\frac{A}{2} = A \cos \left( \frac{2\pi t}{T} \right) \Rightarrow \cos \left( \frac{2\pi t}{T} \right) = \frac{1}{2}
\]
2. Take the arccosine of both sides and solve for $t$:
\[
\cos^{-1} \left( \frac{1}{2} \right) = \frac{2\pi t}{T} \Rightarrow t = \frac{T}{2\pi} \cos^{-1} \left( \frac{1}{2} \right)
\]
3. Substitute $\cos^{-1} \left( \frac{1}{2} \right) = \pi/3$:
\[
t = \frac{T}{2\pi} \left( \frac{\pi}{3} \right) = \frac{T}{6}
\]

**Insight:** This problem could also have been solved using the sine equation, but two times would have been needed: (1) the time when the wave is at maximum amplitude ($t_1 = \frac{T}{4}$), and (2) the first time after $t_1$ that the amplitude is again equal to $A/2$ ($t_2 = \frac{5T}{4}$). The difference in these two times is $t_2 - t_1 = \frac{3T}{4}$, the same as from the cosine equation.

87. **Picture the Problem:** A solid disk is suspended vertically about a pivot near its rim and oscillates about the vertical position.

**Strategy:** The disk is a physical pendulum. The center of mass is at the center of the disk so the distance from the pivot to the center of mass is $r = A$. The moment of inertia for a solid cylinder pivoted at its rim is $I = \frac{1}{3}mr^2$.

**Solution:** Insert $\ell$ and $I$ into equation 13-21 and simplify:
\[
T = 2\pi \sqrt{\frac{r}{g} \left( \frac{\ell}{I} \right)} = 2\pi \sqrt{\frac{r}{g} \left( \frac{3}{\frac{1}{3}mr^2} \right)} = 2\pi \sqrt{\frac{0.15 \text{ m}}{9.81 \text{ m/s}^2} \left( \frac{3}{\frac{1}{3}} \right)} = 0.95 \text{ s}
\]

**Insight:** A simple pendulum with a length equal to the diameter of the disk would oscillate with a period $T = 1.10 \text{ s}$, which, as expected, is longer than the period of this physical pendulum.

88. **Picture the Problem:** The mass oscillates with simple harmonic motion about its equilibrium position.

**Strategy:** Use the behavior of a mass on a spring to determine the required ratio. When the mass is at its maximum displacement, the total energy is in the potential energy of the spring. As the mass travels toward the equilibrium, the potential energy is converted into kinetic energy. The kinetic energy is maximum as the mass passes through the equilibrium point. When the mass is at half the maximum displacement, part of the energy is potential and the rest is kinetic. Write the kinetic energy and potential energy at half of the maximum displacement as fractions of the total energy and then determine the ratio $K/U$.

**Solution:** 1. Write the total energy in terms of the maximum amplitude:
\[
E = \frac{1}{2}kA^2
\]
2. Write the potential energy at half maximum in terms of the total energy:
\[
U = \frac{1}{2}k \left( \frac{A}{2} \right)^2 = \frac{1}{4} \left( \frac{1}{2}kA^2 \right) = \frac{1}{4}E
\]
3. Write the kinetic energy at half maximum displacement in terms of the total energy:
\[
K = E - U = E - \frac{1}{4}E = \frac{3}{4}E
\]
4. Find the ratio of kinetic energy to potential energy:
\[
\frac{K}{U} = \frac{\frac{3}{4}E}{\frac{1}{4}E} = 3
\]

**Insight:** At half the maximum distance most of the energy is kinetic energy. This is because the potential energy is a function of the displacement squared.
89. **Picture the Problem:** The mass slides across the floor toward the spring with an initial velocity and kinetic energy. Upon contact with the spring, the block slows down as the kinetic energy is converted to potential energy of the spring. The block comes to rest when all of the initial kinetic energy is spring potential energy.

**Strategy:** Use the conservation of energy to equate the initial kinetic energy of the mass to the final potential energy of the spring to solve for the compression distance. The stopping time is equal to one-quarter of a period of the mass and spring.

**Solution:**

1. (a) Set the initial kinetic energy equal to the final potential energy:

\[
\frac{1}{2}mv^2 = \frac{1}{2}kA^2
\]

2. Solve for the amplitude (or maximum displacement):

\[
A = \sqrt{\frac{mv^2}{k}} = \sqrt{\frac{(0.363 \text{ kg})(1.24 \text{ m/s})^2}{44.5 \text{ N/m}}} = 0.112 \text{ m}
\]

3. (b) Write the time as one-quarter period:

\[
t = \frac{1}{4}T = \frac{\pi}{2} \sqrt{\frac{m}{k}} = \frac{\pi}{2} \sqrt{\frac{0.363 \text{ kg}}{44.5 \text{ N/m}}} = 0.142 \text{ s}
\]

**Insight:** The distance the spring compresses is proportional to the initial speed of the mass. However, the stopping time is independent of the speed of the mass. You can solve part (b) with Newton’s Second Law, but it requires calculus because the force and acceleration are not constant.

90. **Picture the Problem:** When the barge is slightly displaced downward (or upward) in the water, the buoyant force accelerates the barge back through the equilibrium height, causing the barge to bob up and down. This motion can be treated as a damped harmonic oscillation.

**Strategy:** Solve the damped amplitude equation, setting the time as 5.0 minutes (=300 s) and the ratio of amplitudes \(\frac{A_0}{A}\) as equal to 2, to find the damping constant.

**Solution:**

1. Write out the amplitude equation and isolate the exponential:

\[
A = A_0 e^{-bt/2m} \Rightarrow \frac{A}{A_0} = e^{-bt/2m}
\]

2. Take the natural logarithm of both sides and solve for the damping constant:

\[
\ln\left(\frac{A}{A_0}\right) = \frac{-bt}{2m}
\]

\[
b = \left(\frac{2m}{t}\right) \ln\left(\frac{A_0}{A}\right) = \left[\frac{2\left(2.44 \times 10^3 \text{ kg}\right)}{3.0 \times 10^3 \text{ s}}\right] \ln 2 = 1100 \text{ kg/s}
\]

**Insight:** The damping rate decreases the amplitude of the oscillation by a factor of two every five minutes. Therefore if the barge were to oscillate with an initial amplitude of one foot, it would take about 20 minutes before the oscillations were less than an inch in amplitude.
91. **Picture the Problem**: The figure shows the $x$-versus-$t$ graph.

**Strategy**: The maximum speed is the product of the amplitude and the angular speed, where the angular speed is $2\pi$ divided by the period. Determine the period from the graph and solve for the angular speed and the maximum speed. The energy is equal to the maximum kinetic energy.

**Solution**: 1. (a) From the graph we can determine that the period is less than $2\pi$, so the angular speed is greater than 1 rad/s, and the maximum speed is greater than 0.50 m/s.

2. (b) Insert the period of 4.0 s to solve for the angular speed:

   \[
   \omega = \frac{2\pi}{T} = \frac{2\pi}{4.0 \text{ s}} = \frac{\pi}{2} \text{ s}^{-1}
   \]

3. Insert the angular speed into the maximum speed equation:

   \[
   v_{\text{max}} = A\omega = (0.5 \text{ m}) \left( \frac{\pi}{2} \text{ s}^{-1} \right) = 0.79 \text{ m/s}
   \]

4. (c) Insert the maximum speed into the kinetic energy equation:

   \[
   E = \frac{1}{2} m v_{\text{max}}^2 = \frac{1}{2} (3.8 \text{ kg})(0.785 \text{ m/s})^2 = 1.2 \text{ J}
   \]

**Insight**: The total energy of the oscillation is proportional to the square of the maximum velocity, which is in turn proportional to the amplitude and frequency. Therefore, the energy of the oscillation, for a constant amplitude, increases as the frequency of oscillation increases.

92. **Picture the Problem**: A spring oscillates with the amplitude and period shown in the figure.

**Strategy**: The maximum force occurs at points of maximum displacement. Calculate the maximum force from the maximum acceleration. The force is zero when the displacement is zero, so use the acceleration equation to find the force at any time.

**Solution**: 1. (a) From the graph find the times of maximum displacement: $t = 1.0 \text{ s}, 3.0 \text{ s}, 5.0 \text{ s}$

2. (b) Write the acceleration in terms of the amplitude and period:

   \[
   F_{\text{max}} = m a_{\text{max}} = m A \omega^2 = m A \left( \frac{2\pi}{T} \right)^2
   \]

3. Numerically solve for the maximum force:

   \[
   F_{\text{max}} = (3.8 \text{ kg})(0.50 \text{ m}) \left( \frac{2\pi}{4.0 \text{ s}} \right)^2 = 4.7 \text{ N}
   \]

4. (c) From the graph find the times when the displacement (and therefore the acceleration) is zero:

   \[
   t = 0, 2.0 \text{ s}, 4.0 \text{ s}, 6.0 \text{ s}
   \]

5. (d) Write out the acceleration as a function of time in terms of the amplitude and period:

   \[
   a = -a_{\text{max}} \sin(\omega t) = -A \left( \frac{2\pi}{T} \right)^2 \sin \left( \frac{2\pi t}{T} \right)
   \]

6. Multiply acceleration by mass:

   \[
   F = ma = -mA \left( \frac{2\pi}{T} \right)^2 \sin \left( \frac{2\pi t}{T} \right)
   \]

7. Numerically evaluate the force at $t = 0.5 \text{ s}$:

   \[
   F = -(3.8 \text{ kg})(0.5 \text{ m}) \left( \frac{2\pi}{4.0 \text{ s}} \right)^2 \sin \left( \frac{2\pi (0.5 \text{ s})}{4.0 \text{ s}} \right) = -3.3 \text{ N}
   \]

**Insight**: The force at $t = 0.50 \text{ s}$ could alternatively be obtained by setting the ratio of the displacement and maximum displacement equal to the ratio of the force and the maximum force, $x/A = F/m a_{\text{max}}$, and solving for the force.
93. **Picture the Problem**: A crow lands on a branch and it bobs up and down like a mass on a spring. When an eagle lands on the same branch the period of the motion will be slower because the eagle is more massive.

**Strategy**: Use the mass of the crow and the period of oscillation to determine the spring force constant of the branch. Calculate the mass of the eagle from the spring force constant of the branch and the period of the eagle’s oscillation.

**Solution**: 1. (a) Solve the period equation for the spring force constant:

\[
T = 2\pi \sqrt{\frac{m}{k}} \Rightarrow k = \left(\frac{2\pi}{T}\right)^2 m
\]

2. Insert the mass and period:

\[
k = \left(\frac{2\pi}{1.5 \text{ s}}\right)^2 (0.45 \text{ kg}) = 7.9 \text{ N/m}
\]

3. (b) Solve the period equation for the mass:

\[
T = 2\pi \sqrt{\frac{m}{k}} \Rightarrow m = \left(\frac{T}{2\pi}\right)^2 k
\]

4. Insert the spring force constant and the period:

\[
m = \left(\frac{4.8 \text{ s}}{2\pi}\right)^2 (7.9 \text{ N/m}) = 4.6 \text{ kg}
\]

**Insight**: Even though the amplitudes of oscillation between the crow and the eagle could have been different, they do not affect the period of motion. Therefore it is possible to use the oscillation of the branch in measuring the mass of the eagle.

94. **Picture the Problem**: The figure shows two identical masses attached to a spring. The spring oscillates in periodic motion. When the spring is at maximum displacement (speed is zero) the string breaks, causing one of the masses to drop. The spring oscillation then continues with only one mass.

**Strategy**: The equilibrium point (where the net force is zero, or when the spring force equals the weight) will move upward when one of the masses is removed. The remaining mass is then a farther distance from the equilibrium point than before, and the amplitude of the motion is larger. Let \(x = 0\) correspond with the unstretched position of the spring. The new amplitude will be the difference between the position \(x_{\text{break}}\) of the remaining mass at the instant the string breaks and the new location \(x_{\text{eq, new}}\) of the equilibrium position.

**Solution**: 1. Find the equilibrium position for the two masses:

\[
\sum F_y = k x_{\text{eq, old}} - 2mg = 0 \Rightarrow x_{\text{eq, old}} = \frac{2mg}{k}
\]

2. Find the equilibrium position for the single mass:

\[
\sum F_y = k x_{\text{eq, new}} - mg = 0 \Rightarrow x_{\text{eq, new}} = \frac{mg}{k}
\]

3. Find the new amplitude:

\[
A_{\text{new}} = x_{\text{break}} - x_{\text{eq, new}} = \left(x_{\text{eq, old}} + A\right) - x_{\text{eq, new}} = \frac{2mg}{k} + A - \frac{mg}{k} = A + \frac{mg}{k}
\]

**Insight**: From an energy point of view, the second mass helped to stretch the spring farther and store potential energy in the spring. When the second mass was removed, that energy was still stored in the spring and produced the larger oscillation amplitude for the remaining mass.
95. **Picture the Problem**: The pendulum bob is free to swing using the full length of the string on the right side of its oscillation. However, when the bob moves to the left side of the oscillation the peg acts as the pivot point, decreasing the effective length of the string.

**Strategy**: The period is the time for the pendulum to complete one full oscillation. With the peg in place consider the oscillation to be in two parts: Half of the oscillation will occur with a shorter string and half with the longer string. Since the period is proportional to the square root of the string length, it takes a shorter time for the oscillation with the peg.

**Solution 1. (a)** The period is less than that without the peg, because half the cycle is speeded up due to the peg’s shortening of the pendulum.

2. (b) Write out the time for the left half of the oscillation:

\[ T_L = \frac{1}{2} \left( 2\pi \sqrt{\frac{L}{g}} \right) = \pi \sqrt{\frac{L}{g}} \]

3. Write out the time for the right half of the oscillation:

\[ T_R = \frac{1}{2} \left( 2\pi \sqrt{\frac{\ell}{g}} \right) = \pi \sqrt{\frac{\ell}{g}} \]

4. Add the two times to get the full period:

\[ T = \pi \sqrt{\frac{\ell}{g}} + \pi \sqrt{\frac{L}{g}} \]

5. (c) Evaluate the period for the specified lengths:

\[ T = \pi \left( \sqrt{\frac{0.25 \text{ m}}{9.81 \text{ m/s}^2}} + \sqrt{\frac{1.0 \text{ m}}{9.81 \text{ m/s}^2}} \right) = 1.5 \text{ s} \]

**Insight**: The period without the peg would be \( T = 2\pi \sqrt{\frac{1.0 \text{ m}}{9.81 \text{ m/s}^2}} = 2.0 \text{ s} \) which is longer than the time with the peg.

96. **Picture the Problem**: When the mass is attached to the vertical spring it stretches the spring by a distance \( L \). When the mass is pulled down slightly farther and released, the mass and spring oscillate vertically.

**Strategy**: Use Hooke’s Law to calculate the spring force constant from the gravitational force and stretch distance. Insert the spring force constant into the period equation to find the relationship between the stretch length and period. This result will be compared with period of a pendulum of the same length.

**Solution**: 1. (a) Set the force of the spring equal to the weight of the mass and solve for the ratio \( \frac{m}{k} \):

\[ F_{spring} = kL = mg \quad \Rightarrow \quad \frac{m}{k} = \frac{L}{g} \]

2. Insert the ratio into equation 13-11:

\[ T = 2\pi \sqrt{\frac{m}{k}} = 2\pi \sqrt{\frac{L}{g}} \]

3. (b) The period of a simple pendulum is given by equation 13-20:

\[ T = 2\pi \sqrt{\frac{L}{g}} \]

**Insight**: The periods of oscillation for these two systems are the same.

97. **Picture the Problem**: A mass is attached to a spring, displaced from equilibrium a distance \( A \), and released. As it moves back and forth in simple harmonic motion, the sum of the potential energy (maximum at maximum displacement) and the kinetic energy (maximum at zero displacement) equals the total energy (a constant).

**Strategy**: Use the conservation of energy to determine the speed at any location.

**Solution**: 1. Write the conservation of energy equation:

\[ K + U = E \]

\[ \frac{1}{2} mv^2 + \frac{1}{2} kx^2 = \frac{1}{2} kA^2 \]

2. Solve the equation for the speed:

\[ mv^2 = k \left( A^2 - x^2 \right) \quad \Rightarrow \quad v = \sqrt{\frac{k}{m} \left( A^2 - x^2 \right)} \]
3. Write in terms of the angular speed using $\omega = \frac{k}{m}$: $v = \omega \sqrt{A^2 - x^2}$

**Insight:** This final equation could be rewritten as: $x^2 + \left(\frac{v}{\omega}\right)^2 = A^2$. This equation is reminiscent of either the Pythagorean theorem or the equation of a circle. In fact, sometimes periodic motion is described as circular motion in phase space (a graph in which the $x$-direction is the displacement, and the $y$-direction represents velocity).

98. **Picture the Problem:** The image shows a physical pendulum. It is made of a rod of length $L$ with masses $m_2 < m_1$ affixed on each end. The rod is pivoted at the center.

**Strategy:** To calculate the period of a physical pendulum, first find the distance $\ell$ from the center of mass to the pivot and the moment of inertia $I$. These can then be substituted into the physical pendulum equation (equation 13-21) to find the period.

**Solution:**
1. Find the center of mass of the pendulum:
   $$\ell = \frac{\left(\frac{1}{2}\right)m_1 + \left(-\frac{1}{2}\right)m_2}{m_1 + m_2} = \frac{L(m_1 - m_2)}{2(m_1 + m_2)}$$

2. Find the moment of inertia of the pendulum:
   $$I = \left(m_1 + m_2\right) \left(\frac{L}{2}\right)^2$$

3. Simplify the term $\sqrt{\frac{I}{m\ell^2}}$:
   $$\sqrt{\frac{I}{m\ell^2}} = \sqrt{\frac{(m_1 + m_2)\left(\frac{1}{2}\right)^2}{(m_1 + m_2)\left(\frac{1}{2}\right)^2}} = \frac{m_1 + m_2}{m_1 - m_2}$$

4. Determine the period of the physical pendulum:
   $$T = 2\pi \sqrt{\frac{\ell}{g \sqrt{I/m\ell^2}}} = 2\pi \left(\frac{L}{2g(m_1 + m_2)}\right) \frac{m_1 + m_2}{m_1 - m_2} = \pi \sqrt{\frac{2L(m_1 + m_2)}{g(m_1 - m_2)}}$$

**Insight:** In the limit that $m_2 \to 0$, the period equation reduces to $T = 2\pi \sqrt{L/g}$, which is the period of a simple pendulum of length $L/2$. In the limit that $m_2 \to m_1$, the period goes to infinity. When the masses are equal, the rod is symmetric, and it does not oscillate.

99. **Picture the Problem:** The image shows a pencil sitting in a hollow cylinder attached to a speaker. As the frequency of oscillation of the speaker increases beyond a certain value, the pencil begins to bounce up and down in the cylinder.

**Strategy:** Use equation 13-9 to describe the acceleration experienced by the speaker as a function of its frequency and the amplitude of its vibration.

**Solution:**
1. (a) At low frequencies the maximum acceleration of the speaker is less than $g = 9.81 \text{ m/s}^2$, so that the pencil stays in contact with the tube at all times. However, at higher frequencies the acceleration exceeds that due to gravity. The pencil then rattles because it loses contact with the speaker over some parts of the cycle.

2. (b) Set $a_{\text{max}}$ equal to $g$:
   $$a_{\text{max}} = g$$

3. Write $a_{\text{max}}$ in terms of $A$ and $\omega$:
   $$A\omega^2 = g$$

4. Write $\omega$ in terms of frequency:
   $$A(2\pi f)^2 = g$$

5. Solve for the frequency:
   $$f = \frac{1}{2\pi} \sqrt{\frac{g}{A}}$$

**Insight:** At this frequency the pencil will begin to rattle as it loses contact with the cylinder. This equations shows that the pencil will lose contact at lower frequencies if the speaker oscillates with greater amplitude.
100. **Picture the Problem:** The cricket chirps at a rate that is governed by the air temperature (Dolbear’s law).

**Strategy:** Use the slope of the graph (1.0 chirp/13 s/°F) to determine the additional number of chirps ΔN that correspond to a 10°F rise in temperature.

**Solution:** Use the slope of the graph to find ΔN:

\[
\Delta N = \text{slope} \times \Delta T = \left(1.0 \frac{\text{chirp}}{13 \text{ s/°F}}\right)(10 \text{ °F}) = 10 \text{ chirp}/13 \text{ s}
\]

**Insight:** A 13°F rise in temperature will produce an additional 13 chirps/13 s, or an extra chirp per second.

101. **Picture the Problem:** The cricket chirps at a rate that is governed by the air temperature (Dolbear’s law).

**Strategy:** Solve Dolbear’s law \(N = T - 39\) for \(T\) using the given number of chirps \(N\).

**Solution:** Solve Dolbear’s law for \(T\):

\[
T = N + 39 = \frac{35 + 39}{1 \text{ chirp/13 s/°F}} = 74 \text{ °F}
\]

**Insight:** We can use the result of problem 100 to conclude that a temperature of 84°F will produce 45 chirps in 13 s.

102. **Picture the Problem:** The cricket chirps at a rate that is governed by the air temperature.

**Strategy:** The frequency at any temperature can be obtained by dividing the number of chirps \(N = T - 39\) by 13 seconds.

**Solution:** Write the frequency as a function of temperature:

\[
f = \frac{N}{t} = \frac{T - 39}{13 \text{ s}} = \left(68 \text{ °F}\right)\left(1 \text{ chirp/13 s/°F}\right) - 39 \text{ chirps} = 2.2 \text{ Hz}
\]

**Insight:** Many wind-up alarm clocks tick at 2 Hz, about the same frequency as this rapidly chirping cricket.

103. **Picture the Problem:** The cricket chirps at a rate that is governed by the air temperature.

**Strategy:** The frequency at any temperature can be obtained by dividing the number of chirps \(N = T - 39\) by 13 seconds. Find the total number of chirps by multiplying the average frequency times 12 minutes (720 s).

**Solution:**

1. Write the frequency as a function of temperature:

\[
f = \frac{N}{t} = \frac{T - 39}{13 \text{ s}}
\]

2. Find the frequency at 75°F:

\[
f_{75} = \frac{75 - 39}{13 \text{ s}} = 2.77 \text{ Hz}
\]

3. Find the frequency at 63°F:

\[
f_{63} = \frac{63 - 39}{13 \text{ s}} = 1.85 \text{ Hz}
\]

4. Calculate the average frequency:

\[
\frac{2.77 \text{ Hz} + 1.85 \text{ Hz}}{2} = 2.31 \text{ Hz}
\]

5. Multiply the average frequency by the total time:

\[
N = f \cdot t_{\text{total}} = \left(2.31 \text{ Hz}\right)(12 \text{ min})\left(\frac{60 \text{ s}}{\text{min}}\right) = 1700 \text{ chirps}
\]

**Insight:** The method of averaging the frequencies will only work when the temperature increases uniformly. This is similar to the case where an object increases its speed at a uniform rate; the average speed is simply \(v_{\text{ave}} = \frac{1}{2}(v_i + v_f)\).
104. **Picture the Problem**: The 747 airliner is moving up and down with an amplitude of \( A = 30.0 \text{ m} \) relative to the normal horizontal flight path.

**Strategy**: The maximum acceleration is inversely related to the period. Use equations 13-5 and 13-9 to find the period.

**Solution**: 1 (a) In order to decrease the maximum acceleration we must **increase** the period.

2. (b) Write \( a_{\text{max}} \) in terms of the period:

\[
a_{\text{max}} = A \left( \frac{2\pi}{T} \right)^2
\]

3. Solve the expression from step 2 for the period:

\[
T = 2\pi \sqrt{\frac{A}{a_{\text{max}}}} = 2\pi \sqrt{\frac{30.0 \text{ m}}{9.81 \text{ m/s}^2}} = 11 \text{ s}
\]

**Insight**: As predicted the period of 11 s is longer than the period of 8.2 s given in Example 13-3.

105. **Picture the Problem**: The sketch shows the mass moving toward the spring with an initial speed of \( v_0 \). The mass then collides with the spring and compresses it a distance \( A' \) before coming to rest.

**Strategy**: Use conservation of energy to determine the compression distance as a function of the spring force constant. Since the kinetic energy is unchanged, the same potential energy will be necessary to stop the mass. The potential energy is proportional to the square of the amplitude. The time to stop is one-quarter of a period.

**Solution 1. (a)** Doubling the spring force constant will cause the square of the amplitude to be cut in half. Therefore the amplitude will **decrease by a factor of \( \sqrt{2} \)**.

2. (b) Set the maximum kinetic energy equal to the maximum potential energy:

\[
\frac{1}{2} m v_0^2 = \frac{1}{2} k' (A')^2
\]

3. Solve the energy equation for the amplitude:

\[
A' = v_0 \sqrt{\frac{m}{k'}}
\]

4. Set \( k' = 2k \) and solve for \( A' \) as a function of \( A \).

\[
A' = v_0 \sqrt{\frac{m}{2k}} = \frac{1}{\sqrt{2}} \left( v_0 \sqrt{\frac{m}{k}} \right) = \frac{A}{\sqrt{2}}
\]

5. Numerically evaluate the new compression:

\[
A' = \frac{0.0835}{\sqrt{2}} = 0.0835 \text{ m} = 5.90 \text{ cm}
\]

6. (c) Determine one-quarter period:

\[
t = \frac{T}{4} = \frac{\pi}{2} \sqrt{\frac{m}{k}}
\]

7. Insert the numeric values:

\[
t = \frac{\pi}{2} \sqrt{\frac{0.980 \text{ kg}}{490 \text{ N/m}}} = 0.0702 \text{ s}
\]

**Insight**: The stiffer spring will cause the mass to come to rest in a shorter distance and over a shorter time period.
106. **Picture the Problem:** The sketch shows the mass moving toward the spring with an initial speed of \( v_0 > v_0 \), where \( v_0 \) is from the original example. The mass then collides with the spring and compresses it a distance \( A' \) before coming to rest.

**Strategy:** The spring is in contact with the block for one-half of an oscillation period. The period is determined by the mass and the spring force constant.

**Solution 1. (a)** The period does not depend on the initial speed of the block, so the total time of contact is the same.

2. (b) Write the time for half a period using equation 13-11:

\[
t = \frac{T}{2} = \pi \sqrt{\frac{m}{k}} = \pi \sqrt{\frac{0.980 \text{ kg}}{245 \text{ N/m}}} = 0.199 \text{ s}
\]

**Insight:** This is the same time that the slower mass was in contact with the spring. The higher speed will cause the spring to compress a greater distance. However, it compresses the spring at a faster rate. The faster rate and longer distance combine so that the time remains constant.